Parameterized Synthesis of Self-Stabilizing Protocols in Symmetric Rings

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Iowa State University, USA[3]

Presentation outline

Motivation

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Formal Specification

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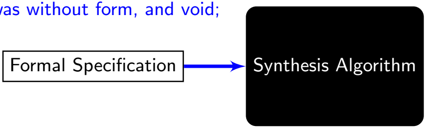

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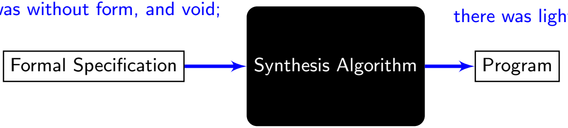

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- It is particularly useful to deal with small but *intricate components* of a system, e.g., concurrent/distributed algorithms.

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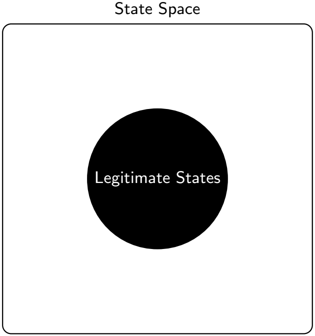

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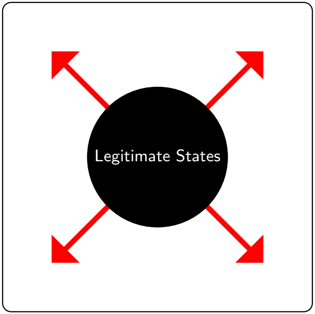

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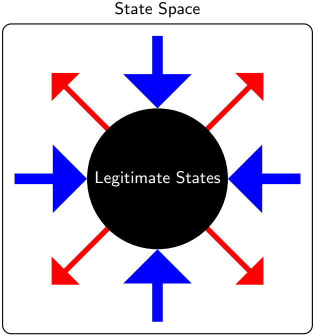

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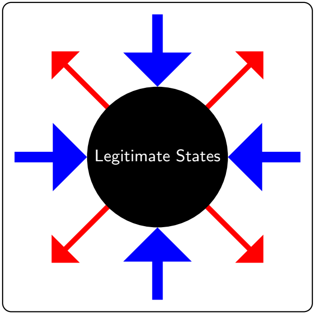

- *Closure.* Once the system reaches a legitimate state, it remains in this set thereafter in the absence of new faults.

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- *Proof* of self-stabilization is often much more complex than what it initially seems like.
- *Dijkstra* himself published the proof of correctness of his seminal 3-state machine solution *12 years* later.
- *Program synthesis* can play a prime role in designing and reasoning about the correctness of *self-stabilizing* algorithms.

Related Work

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Problem

All these approaches can synthesize only a *fixed* number of processes.
Parameterized synthesis is an *undecidable* problem.

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- *Tight cutoffs* for the *closure* and *deadlock-freedom* properties.
- A *sufficient condition* for *convergence* that can be efficiently checked on an over-approximated finite system.
- A scalable *counterexample-guided synthesis* technique.

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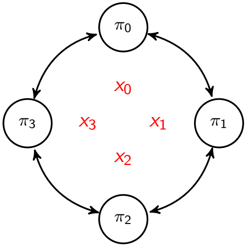

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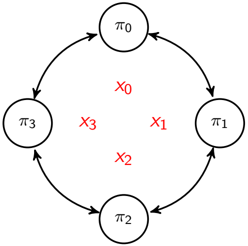

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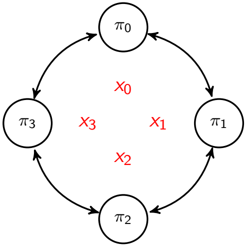

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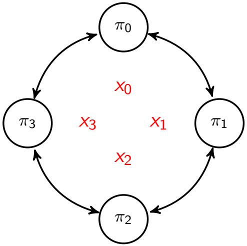

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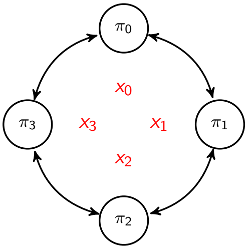

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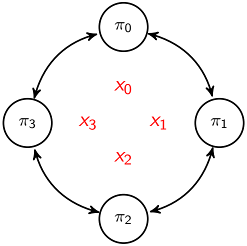

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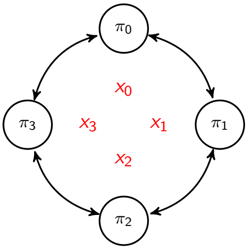

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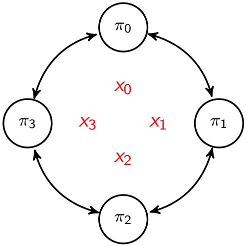

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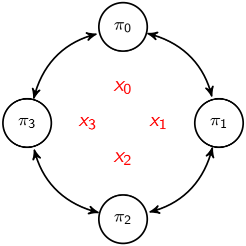

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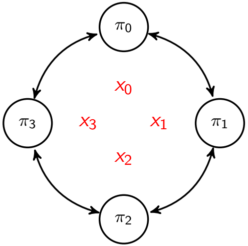

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Such transitions create an equivalence class, called a *group*.

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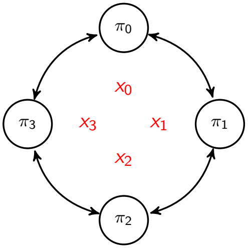

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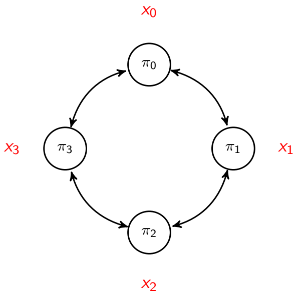

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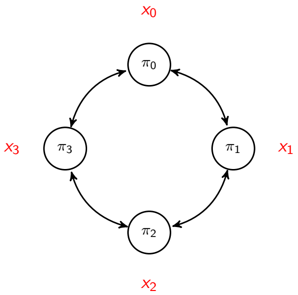

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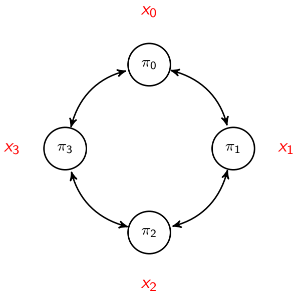

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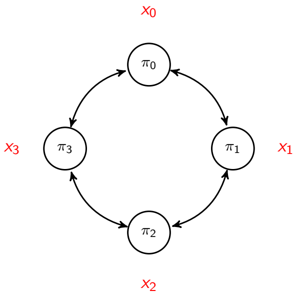

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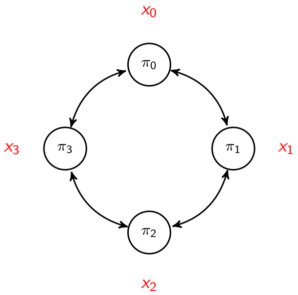

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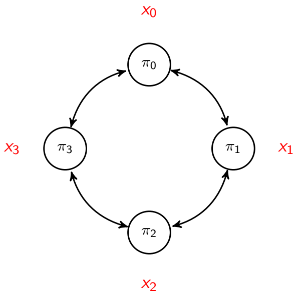

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- $W_{\mathcal{T}}$ is a mapping $\{0 \ldots |P_{\mathcal{T}}| - 1\} \rightarrow 2^V$ from a process index to its write-set, such that $W_{\mathcal{T}}(i) \subseteq R_{\mathcal{T}}(i)$, for all i $(0 \leq i \leq |P_{\mathcal{T}}| - 1)$.

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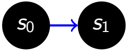

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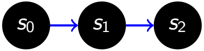

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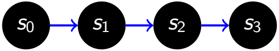

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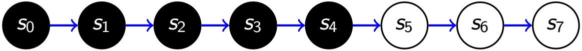

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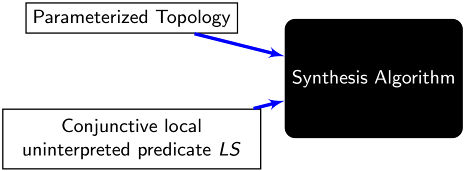

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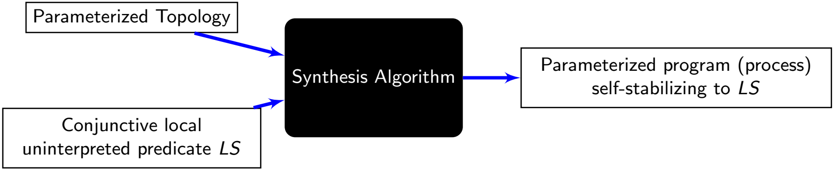

Presentation outline

The Notion of Cutoff

For a given parameterized topology and a property, a *cutoff* is a natural number c, such that for any given process π and a locally defined LS the following holds:

> $\mathcal{D}_n = \mathcal{T}_n^\pi$ satisfies the property wrt. LS for all $n \in \mathbb{N}$ iff $\mathcal{D}_i = \mathcal{T}_i^\pi$ satisfies the property wrt. LS for all $i \in [1 \ldots c]$.

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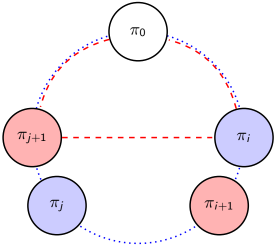

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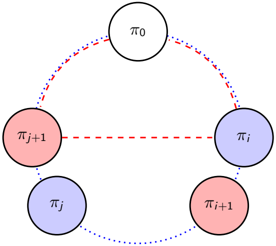

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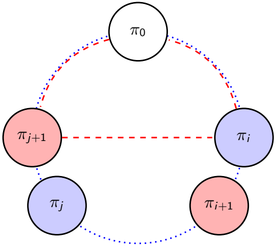

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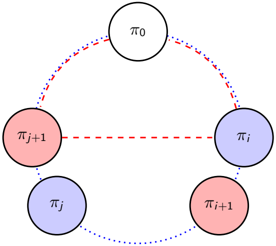

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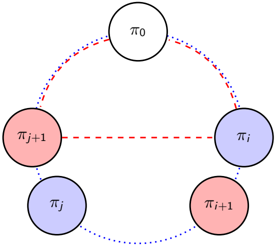

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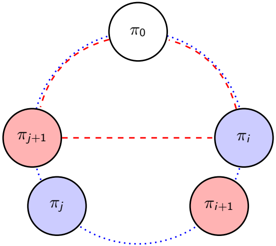

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- We can repeat the removal of processes until we arrive at $c = l^2 + 1$.

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All of the cutoffs are tight under their respective assumptions.

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We check whether there is a loop that starts and ends in the same local state for an arbitrary process.

If we can show that this is not possible, then certainly no global loop is possible.

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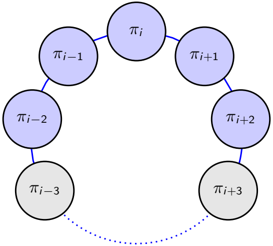

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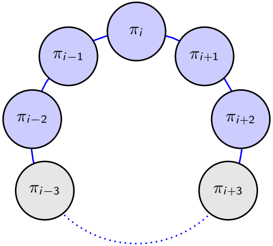

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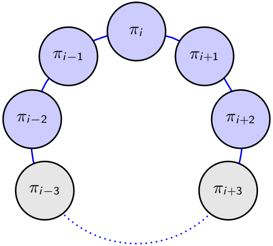

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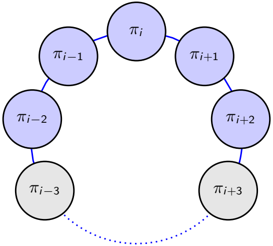

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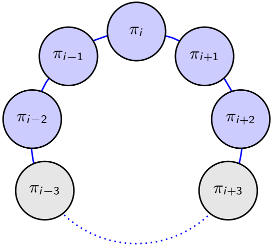

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- These two processes *over-approximate* the possible behavior of all other processes.
- The *precision of the abstraction* can be refined by increasing the number of processes that behave according to the protocol, or by including the local state of additional processes into S.

General Cutoff Result

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Theorem

Let $\mathcal{T}_1, \mathcal{T}_2, \ldots$ be a parameterized ring topology, π a process, and let LS be locally defined by LS_i. Let c_1 and c_2 be cutoffs for closure and deadlock detection wrt. LS, respectively. If

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then every instance *of the parameterized program $\mathcal{D}_1 = \mathcal{T}_1^\pi, \mathcal{D}_2 = \mathcal{T}_2^\pi, \ldots$ is self-stabilizing to LS.*

Presentation outline

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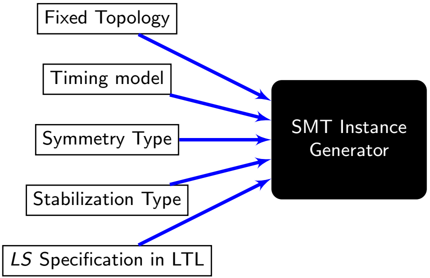
$\boxed{LS \text{ Specification in LTL}}$

[*] F. Faghih and B. Bonakdarpour: *ASSESS: A Tool for Synthesizing Self-Stabilizing Protocols*, SSS 2017.

SMT-based Synthesis – The Tool ASSESS [*]



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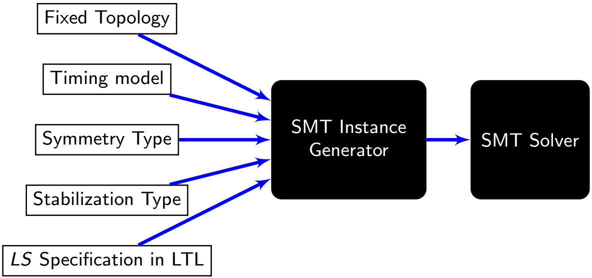

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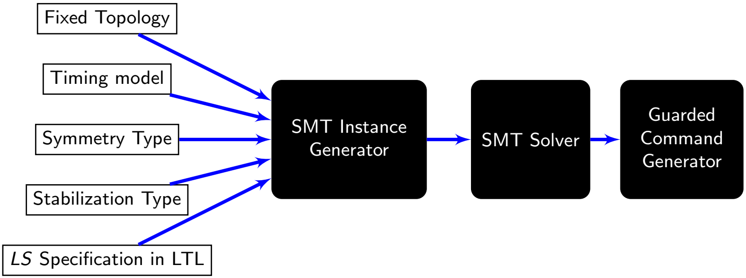

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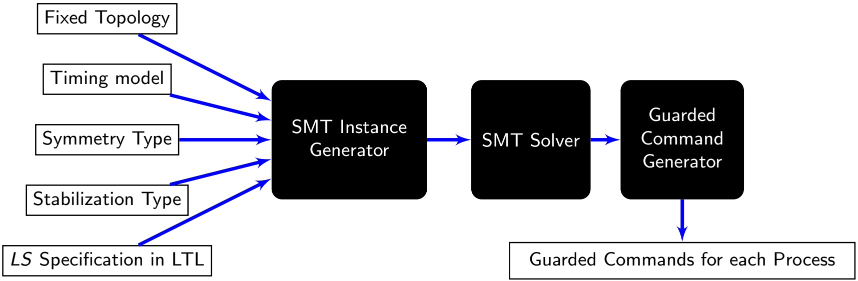

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Counterexample-guided Synthesis

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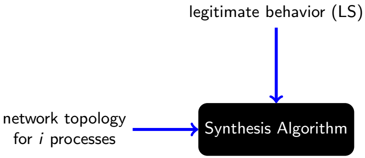
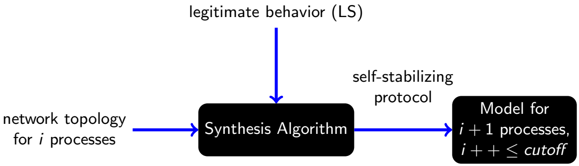
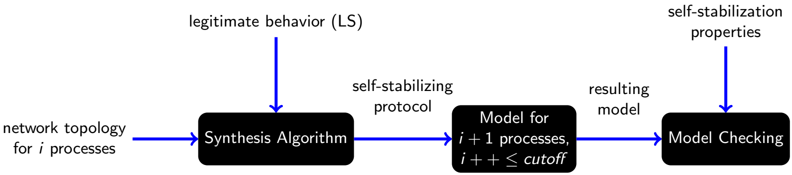


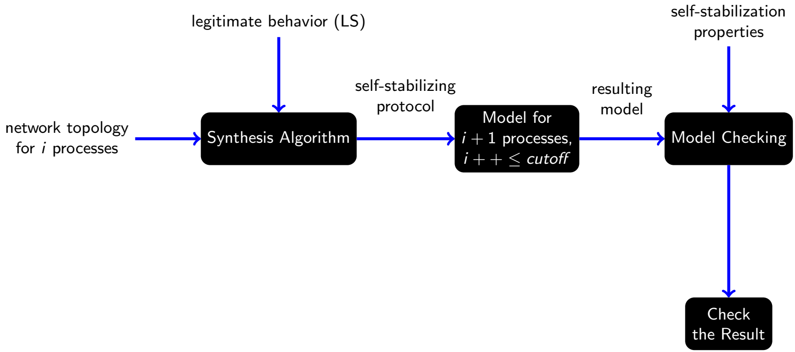
Counterexample-guided Synthesis



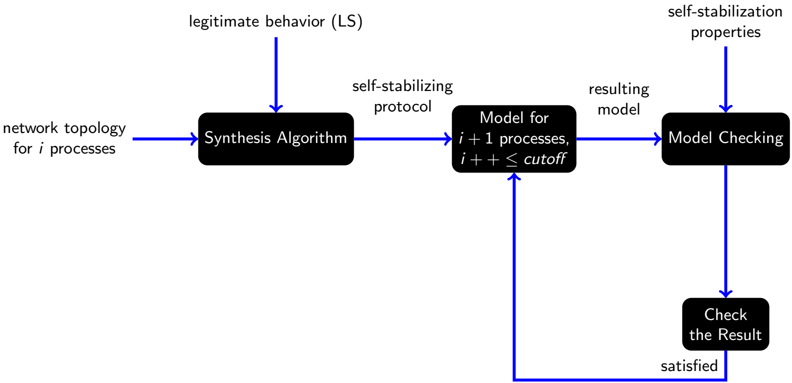
Counterexample-guided Synthesis



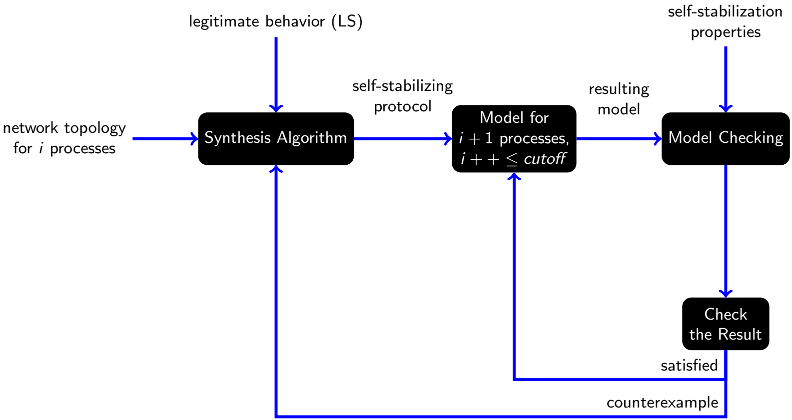
Counterexample-guided Synthesis



Counterexample-guided Synthesis



Counterexample-guided Synthesis



Experimental Results

Problem	**cutoff #**	**Heuristic**	**Synthesis Time**	**Model Checking Time**
Three Coloring	10	Local *LS*	7m 3sec	16 msec
Three Coloring	10	Progress	9m 5sec	16 msec
One-Bit MM	5	Local *LS*	1m 48sec	27 msec
One-Bit MM	5	Progress	1m 44sec	33 msec
Maximal Matching	10	Local *LS*	7m 59sec	36 msec
Maximal Matching	10	Progress	4m 57sec	37 msec
Maximal Independent Set	5	Local *LS*	10sec	18 msec

Experimental Results

Guarded commands of the *1-bit maximal matching* problem:

$$\begin{array}{rrcl} \pi_i : & (x_i = \mathit{false}) \wedge (x_{(i+1)} = \mathit{false}) \wedge (x_{(i-1)} = \mathit{false}) & \longrightarrow & x_i := \mathit{true} \\ & (x_i = \mathit{true}) \wedge (x_{(i+1)} = \mathit{true}) & \longrightarrow & x_i := \mathit{false} \end{array}$$

Experimental Results

Guarded commands of the *1-bit maximal matching* problem:

$$
\begin{aligned}
\pi_i : \quad (x_i = \mathit{false}) \wedge (x_{(i+1)} = \mathit{false}) \wedge (x_{(i-1)} = \mathit{false}) \quad &\longrightarrow \quad x_i := \mathit{true} \\
(x_i = \mathit{true}) \wedge (x_{(i+1)} = \mathit{true}) \quad &\longrightarrow \quad x_i := \mathit{false}
\end{aligned}
$$

The *interpretation function* for match_i is the following:

$$
\begin{aligned}
\mathit{match}_i : \quad (x_i = \mathit{true}) \wedge (x_{(i+1)} = \mathit{true}) \wedge (x_{(i-1)} = \mathit{true}) \quad &\mapsto \quad l \\
(x_{(i+1)} = \mathit{false}) \wedge (x_{(i-1)} = \mathit{false}) \quad &\mapsto \quad l \\
(x_i = \mathit{true}) \wedge (x_{(i+1)} = \mathit{false}) \wedge (x_{(i-1)} = \mathit{true}) \quad &\mapsto \quad r \\
(x_i = \mathit{false}) \wedge (x_{(i+1)} = \mathit{true}) \quad &\mapsto \quad r \\
(x_i = \mathit{true}) \wedge (x_{(i+1)} = \mathit{true}) \wedge (x_{(i-1)} = \mathit{false}) \quad &\mapsto \quad n \\
(x_i = \mathit{false}) \wedge (x_{(i+1)} = \mathit{false}) \wedge (x_{(i-1)} = \mathit{true}) \quad &\mapsto \quad n
\end{aligned}
$$

Presentation outline

Summary

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- To scale up to the cutoff point, we introduced an iterative loop of synthesis and verification guided by counterexamples.
- We synthesized (in less then *10 minutes*) parameterized:
  - Three coloring
  - Maximal matching
  - Maximal independent set

Future Work

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- Other *topologies* (grid, tree, torus, line, etc).

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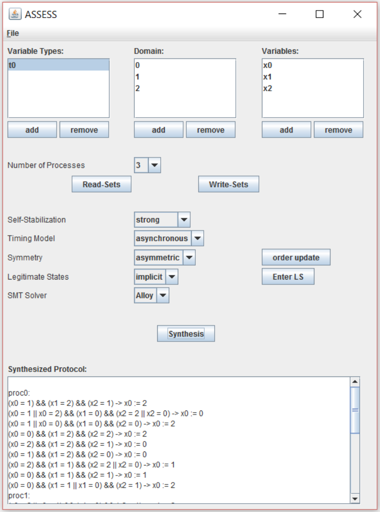
Future Work

- Other *topologies* (grid, tree, torus, line, etc).
- *Asymmetric* netowrk
- *Dynamic* networks
- Protocol *live* in the set of legitimate states.

Commercial 1!

Please download and use ASSESS!

http://web.cs.iastate.edu/~borzoo/assess



Commercial 2!

I am looking for Ph.D. students and postdocs!

Thank you!