Linear rendezvous with asymmetric clocks.

Ryan Killick[1] Evangelos Kranakis[1] Jurek Czyzowicz[2]

[1]School of Computer Science
Carleton University
Ottawa, Canada

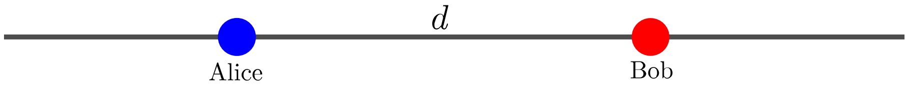
[2]Département d'informatique
Université du Québec en Outaouais
Gatineau, Canada

OPODIS2018, Dec. 17 – 19, 2018

Table of Contents

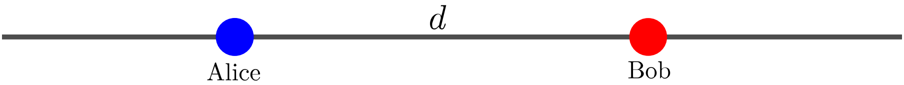
The problem – Symmetric rendezvous

- Two robots placed at arbitrary positions on an infinite line wish to meet...



The problem – Symmetric rendezvous

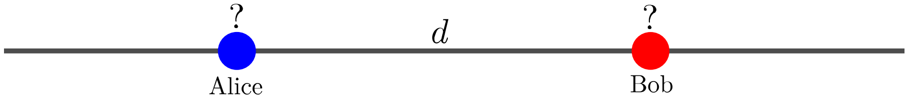
- Two robots placed at arbitrary positions on an infinite line wish to meet...
- The robots are **anonymous** and must both run the same algorithm...



The problem – Symmetric rendezvous

- Two robots placed at arbitrary positions on an infinite line wish to meet...
- The robots are **anonymous** and must both run the same algorithm...

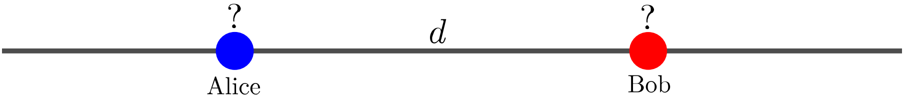
What should the robots do?



The problem – Symmetric rendezvous

- Two robots placed at arbitrary positions on an infinite line wish to meet...
- The robots are **anonymous** and must both run the same algorithm...

What should the robots do?



- Without extra assumptions there is nothing they can do...

robots need to **Break Symmetry**

Related work

- Rendezvous and symmetry breaking

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]
  - ⋮

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]
  - ⋮
- Search on a line [Beck, 1963], [Bellman, 1964], [Baeza Yates et. al., 1993] ...

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]
  - ⋮
- Search on a line [Beck, 1963], [Bellman, 1964], [Baeza Yates et. al., 1993] ...
- Rendezvous and Gathering

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]
  - ⋮
- Search on a line [Beck, 1963], [Bellman, 1964], [Baeza Yates et. al., 1993] ...
- Rendezvous and Gathering
  - Different speeds [Feinerman at. al., 2017], [Bouchard et. al., 2018]

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]
  - ⋮
- Search on a line [Beck, 1963], [Bellman, 1964], [Baeza Yates et. al., 1993] ...
- Rendezvous and Gathering
  - Different speeds [Feinerman at. al., 2017], [Bouchard et. al., 2018]
  - Different orientations/sense of direction [Cohen and Peleg, 2006], [Izumi et. al. 2012], ...

Related work

- Rendezvous and symmetry breaking
  - Labels [De Marco et. al., 2006], [Stachowiak, 2009], ...
  - Tokens [Yu and Yung, 1996], [Kranakis et. al., 2003]
  - Position awareness [Collins et. al., 2010]
  - ⋮
- Search on a line [Beck, 1963], [Bellman, 1964], [Baeza Yates et. al., 1993] ...
- Rendezvous and Gathering
  - Different speeds [Feinerman at. al., 2017], [Bouchard et. al., 2018]
  - Different orientations/sense of direction [Cohen and Peleg, 2006], [Izumi et. al. 2012], ...

Break symmetry using asymmetric speeds and/or clocks!

The model

- Model is synchronous and continuous

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$
- Robots possess clocks to time their movements

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$
- Robots possess clocks to time their movements
  - These clocks may be inconsistent
  - In one unit of "global" time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$
- Robots possess clocks to time their movements
  - These clocks may be inconsistent
  - In one unit of "global" time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times

Different speeds and clocks are sufficient to break symmetry.
Robots do not need to know this!

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$
- Robots possess clocks to time their movements
  - These clocks may be inconsistent
  - In one unit of "global" time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times

Different speeds and clocks are sufficient to break symmetry.
Robots do not need to know this!

- Consider three sub-models
  1. T-Model: $\tau = 1$

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$
- Robots possess clocks to time their movements
  - These clocks may be inconsistent
  - In one unit of "global" time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times

> Different speeds and clocks are sufficient to break symmetry.
> Robots do not need to know this!

- Consider three sub-models
  1. T-Model: $\tau = 1$
  2. D-Model: $v\tau = 1$

The model

- Model is synchronous and continuous
- Robots move with constant speeds
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$
- Robots possess clocks to time their movements
  - These clocks may be inconsistent
  - In one unit of “global” time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times

Different speeds and clocks are sufficient to break symmetry.
Robots do not need to know this!

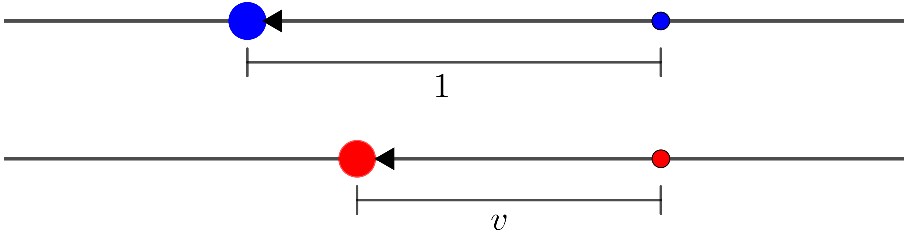
- Consider three sub-models
  1. T-Model: $\tau = 1$
  2. D-Model: $v\tau = 1$
  3. V-Model: $v = 1$

The model: T-Model

- Robot clocks are consistent, $\tau = 1$
- Robots move with different speeds, $v \neq 1$
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$

The model: T-Model

- Robot clocks are consistent, $\tau = 1$
- Robots move with different speeds, $v \neq 1$
  - Alice moves with speed 1
  - Bob moves with speed $v \neq 1$



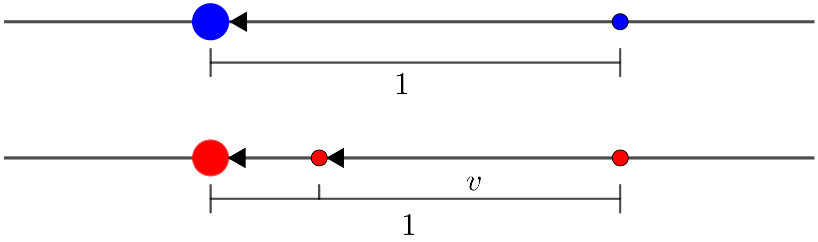
- Robots finish an instruction at the same time but travel different distances

The model: D-Model

- The product $v\tau = 1$
- Robots agree on distance units
  - $v \neq 1$ and $\tau \neq 1$

The model: D-Model

- The product $v\tau = 1$
- Robots agree on distance units
  - $v \neq 1$ and $\tau \neq 1$



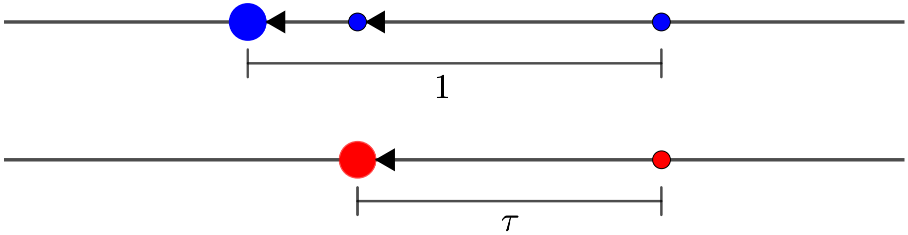
- Robots finish an instruction at different times but travel the same distance

The model: V-Model

- Robots move with the same speed, $v = 1$
- Robot clocks are inconsistent, $\tau \neq 1$
  - In one unit of “global” time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times

The model: V-Model

- Robots move with the same speed, $v = 1$
- Robot clocks are inconsistent, $\tau \neq 1$
  - In one unit of "global" time
    - The clock of Alice ticks once
    - The clock of Bob ticks $\tau \neq 1$ times



- Robots finish an instruction at different times and travel different distances

Reference frames

- A robot's speed and time unit defines a reference frame

Reference frames

- A robot's speed and time unit defines a reference frame
- Can convert between reference frames

Reference frames

- A robot's speed and time unit defines a reference frame
- Can convert between reference frames
  - Trajectory as a (multivalued) function of position

$$t(x) \to \tau t\left(\frac{x \pm d}{v\tau}\right)$$

Reference frames

- A robot's speed and time unit defines a reference frame
- Can convert between reference frames
  - Trajectory as a (multivalued) function of position

$$t(x) \to \tau t \left(\frac{x \pm d}{v\tau} \right)$$

  - Trajectory as a function of time

$$x(t) \to v\tau x \left(\frac{t}{\tau} \right) \pm d$$

Reference frames

- A robot's speed and time unit defines a reference frame
- Can convert between reference frames
  - Trajectory as a (multivalued) function of position

$$t(x) \to \tau t\left(\frac{x \pm d}{v\tau}\right)$$

  - Trajectory as a function of time

$$x(t) \to v\tau x\left(\frac{t}{\tau}\right) \pm d$$

Robots follow different trajectories despite using the same algorithm!

Rendezvous algorithms, 1

Two methods to specify a rendezvous algorithm:

1. Specify a sequence of turning points, X_k

Rendezvous algorithms, 1

Two methods to specify a rendezvous algorithm:

1. Specify a sequence of turning points, X_k
   - Indicates where a robot changes its direction

Rendezvous algorithms, 1

Two methods to specify a rendezvous algorithm:

1. Specify a sequence of turning points, X_k
   - Indicates where a robot changes its direction
   - Move at constant speed between turning points

Rendezvous algorithms, 1

Two methods to specify a rendezvous algorithm:

1. Specify a sequence of turning points, X_k
   - Indicates where a robot changes its direction
   - Move at constant speed between turning points
   - Times at which to turn

$$T_k = X_k + 2\sum_{i=0}^{k-1} X_i$$

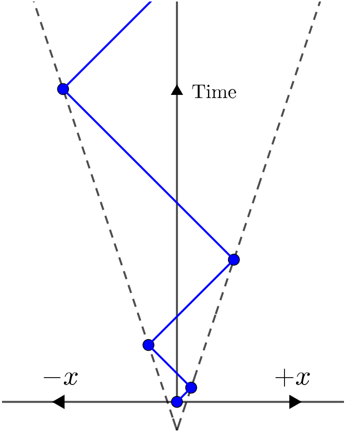
Rendezvous algorithms, 1

Two methods to specify a rendezvous algorithm:

1. Specify a sequence of turning points, X_k
   - Indicates where a robot changes its direction
   - Move at constant speed between turning points
   - Times at which to turn

$$T_k = X_k + 2\sum_{i=0}^{k-1} X_i$$

Turning points define a cone-shaped curve $\mathcal{T}(x)$ in time-position space.



Rendezvous algorithms, 2

Two methods to specify a rendezvous algorithm:

2. Specify the cone $\mathcal{T}(x)$

Rendezvous algorithms, 2

Two methods to specify a rendezvous algorithm:

2. Specify the cone $\mathcal{T}(x)$
   - Turning points induced by $\mathcal{T}(x)$
   - Robots turn when trajectory intersects cone

$$T_k = \mathcal{T}(X_k)$$

Rendezvous algorithms, 2

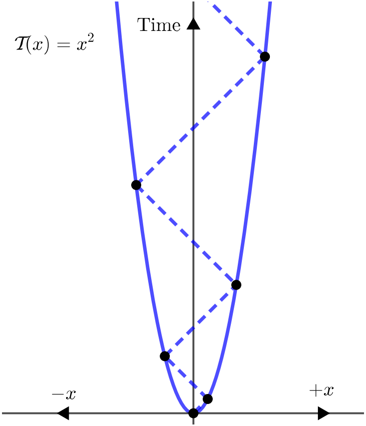
Two methods to specify a rendezvous algorithm:

2. Specify the cone $\mathcal{T}(x)$
   - Turning points induced by $\mathcal{T}(x)$
   - Robots turn when trajectory intersects cone

$$T_k = \mathcal{T}(X_k)$$

Change of reference frame applies to $\mathcal{T}(x)$

$$\mathcal{T}'(x) = \tau \mathcal{T}\left(\frac{x \mp d}{v\tau}\right)$$



Rendezvous algorithms, 3

- Focus on Symmetric, Periodic, and Monotonic (SPM) algorithms

Rendezvous algorithms, 3

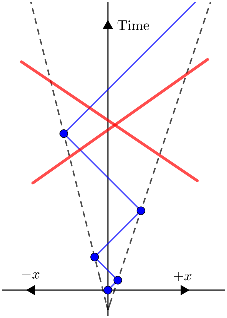
- Focus on Symmetric, Periodic, and Monotonic (SPM) algorithms
- Symmetric
  - The cone $\mathcal{T}(x)$ is left-right symmetric

$$\mathcal{T}(x) = \mathcal{T}(-x)$$

Rendezvous algorithms, 3

- Focus on Symmetric, Periodic, and Monotonic (SPM) algorithms
- Symmetric
  - The cone $\mathcal{T}(x)$ is left-right symmetric

$$\mathcal{T}(x) = \mathcal{T}(-x)$$



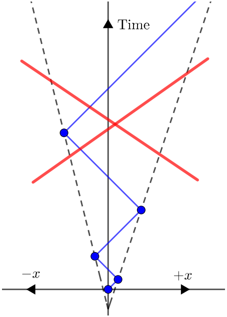
Rendezvous algorithms, 3

- Focus on Symmetric, Periodic, and Monotonic (SPM) algorithms
- Symmetric
  - The cone $\mathcal{T}(x)$ is left-right symmetric

$$\mathcal{T}(x) = \mathcal{T}(-x)$$

- Periodic and Monotonic
  - The turning points alternate sides and are increasing in absolute value

$$\cdots X_{2k-1} < X_{2k+1} < \cdots < 0 < \cdots < X_{2k} < X_{2k+2} < \cdots$$



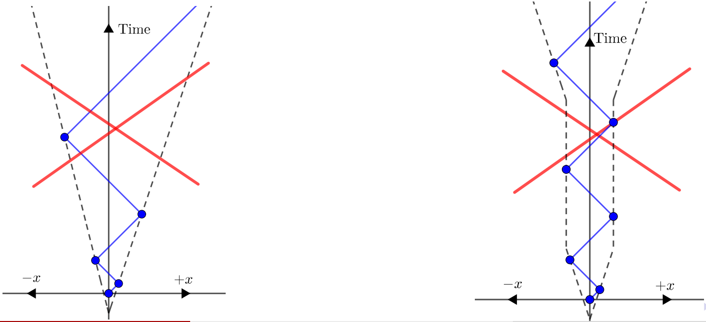
Rendezvous algorithms, 3

- Focus on Symmetric, Periodic, and Monotonic (SPM) algorithms
- Symmetric
  - The cone $\mathcal{T}(x)$ is left-right symmetric

$$\mathcal{T}(x) = \mathcal{T}(-x)$$

- Periodic and Monotonic
  - The turning points alternate sides and are increasing in absolute value

$$\cdots X_{2k-1} < X_{2k+1} < \cdots < 0 < \cdots < X_{2k} < X_{2k+2} < \cdots$$



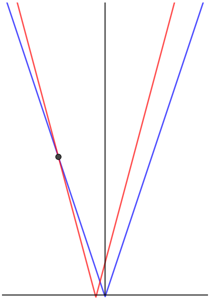
Feasibility

Rendezvous is guaranteed if the cone of one robot contains the other

Feasibility

Rendezvous is guaranteed if the cone of one robot contains the other

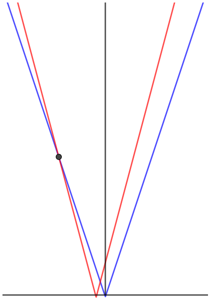
- Containment
  - All left turning points of Alice are to the left of Bob's
  - All right turning points of Alice are to the right of Bob's



Feasibility

Rendezvous is guaranteed if the cone of one robot contains the other

- Containment
  - All left turning points of Alice are to the left of Bob's
  - All right turning points of Alice are to the right of Bob's
- Implies that robot trajectories must intersect



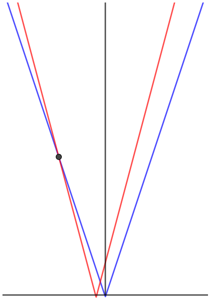
Feasibility

Rendezvous is guaranteed if the cone of one robot contains the other

- Containment
  - All left turning points of Alice are to the left of Bob's
  - All right turning points of Alice are to the right of Bob's
- Implies that robot trajectories must intersect
- Many cones satsify this property
  - If $\mathcal{T}(x)$ is linear

$$\mathcal{T}'(x) = \tau \mathcal{T}\left(\frac{x \mp d}{v\tau}\right) = \frac{1}{v}\mathcal{T}(x \mp d)$$

  - $\mathcal{T}(x) = x^2$



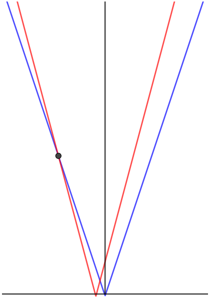
Feasibility

Rendezvous is guaranteed if the cone of one robot contains the other

- Containment
  - All left turning points of Alice are to the left of Bob's
  - All right turning points of Alice are to the right of Bob's
- Implies that robot trajectories must intersect
- Many cones satsify this property
  - If $\mathcal{T}(x)$ is linear

$$\mathcal{T}'(x) = \tau\mathcal{T}\left(\frac{x \mp d}{v\tau}\right) = \frac{1}{v}\mathcal{T}(x \mp d)$$

  - $\mathcal{T}(x) = x^2$
- Search for optimal algorithms
- Optimize the competitive ratio



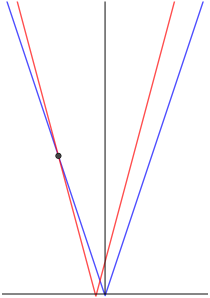
Feasibility

Rendezvous is guaranteed if the cone of one robot contains the other

- Containment
  - All left turning points of Alice are to the left of Bob's
  - All right turning points of Alice are to the right of Bob's
- Implies that robot trajectories must intersect
- Many cones satsify this property
  - If $\mathcal{T}(x)$ is linear

$$\mathcal{T}'(x) = \tau\mathcal{T}\left(\frac{x \mp d}{v\tau}\right) = \frac{1}{v}\mathcal{T}(x \mp d)$$

  - $\mathcal{T}(x) = x^2$
- Search for optimal algorithms
- Optimize the competitive ratio



$$CR_{T,D} := \frac{T_{\mathcal{A}}}{d/|1-v|}$$

$$CR_V := \frac{T_{\mathcal{A}}}{d/|1-\tau|}$$

Table of Contents

T-Model

Robots differ only in their speeds, $\tau = 1$

T-Model

Robots differ only in their speeds, $\tau = 1$

Theorem

Rendezvous in T-Model is equivalent to search for a stationary target at an initial distance $d/(1 - v)$.

T-Model

Robots differ only in their speeds, $\tau = 1$

Theorem

Rendezvous in T-Model is equivalent to search for a stationary target at an initial distance $d/(1-v)$.

- Move to reference frame fixed on Bob
  - Bob appears stationary at an initial distance $d/(1-v)$ from Alice

T-Model

Robots differ only in their speeds, $\tau = 1$

Theorem

Rendezvous in T-Model is equivalent to search for a stationary target at an initial distance $d/(1-v)$.

- Move to reference frame fixed on Bob
  - Bob appears stationary at an initial distance $d/(1-v)$ from Alice
- Can use results known about search on the infinite line...

T-Model

Robots differ only in their speeds, $\tau = 1$

Theorem

Rendezvous in T-Model is equivalent to search for a stationary target at an initial distance $d/(1-v)$.

- Move to reference frame fixed on Bob
  - Bob appears stationary at an initial distance $d/(1-v)$ from Alice
- Can use results known about search on the infinite line...

Theorem

Rendezvous in T-Model is optimally solved with a competitive ratio of 9.

T-Model

Robots differ only in their speeds, $\tau = 1$

Theorem

Rendezvous in T-Model is equivalent to search for a stationary target at an initial distance $d/(1-v)$.

- Move to reference frame fixed on Bob
  - Bob appears stationary at an initial distance $d/(1-v)$ from Alice
- Can use results known about search on the infinite line...

Theorem

Rendezvous in T-Model is optimally solved with a competitive ratio of 9.

- When $\tau = 1$ the robots stay synchronized
  - they arrive to their k^{th} turning points at the same time!

Table of Contents

D-Model

- The product $v\tau = 1$, i.e. equal distance units

D-Model

- The product $v\tau = 1$, i.e. equal distance units
- Reference frame trick does not work

D-Model

- The product $v\tau = 1$, i.e. equal distance units
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times

D-Model

- The product $v\tau = 1$, i.e. equal distance units
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times

Theorem

Rendezvous in D-Model is solved with a competitive ratio of $105/11 \approx 9.55$.

D-Model

- The product $v\tau = 1$, i.e. equal distance units
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times

Theorem

Rendezvous in D-Model is solved with a competitive ratio of $105/11 \approx 9.55$.

- Use the algorithm $X_k = 2^k$

D-Model

- The product $v\tau = 1$, i.e. equal distance units
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times

Theorem

Rendezvous in D-Model is solved with a competitive ratio of $105/11 \approx 9.55$.

- Use the algorithm $X_k = 2^k$
  - Implies the cone $\mathcal{T}(x) = 3|x| - 2$ for Alice
  - Cone of Bob: $\mathcal{T}'(x) = \tau\mathcal{T}\left(\frac{x \mp d)}{v\tau}\right) = \frac{3}{v}|x \mp d| - 2\tau$
  - $v \neq 1$ implies a "quick" rendezvous

Table of Contents

V-Model

- Robots differ only in their time units, $v = 1$

V-Model

- Robots differ only in their time units, $v = 1$
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times

V-Model

- Robots differ only in their time units, $v = 1$
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times
- Cannot use the algorithm $X_k = 2^k$
  - Cone of Alice: $\mathcal{T}(x) = 3|x| - 2$
  - Cone of Bob: $\mathcal{T}'(x) = 3|x \mp d| - 2\tau$
  - These may not contain each other

V-Model

- Robots differ only in their time units, $v = 1$
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times
- Cannot use the algorithm $X_k = 2^k$
  - Cone of Alice: $\mathcal{T}(x) = 3|x| - 2$
  - Cone of Bob: $\mathcal{T}'(x) = 3|x \mp d| - 2\tau$
  - These may not contain each other
- Try the algorithm $\mathcal{T} = x^2$
  - $\mathcal{T}'(x) = \tau \left(\frac{x \mp d}{v\tau}\right)^2 = \frac{1}{\tau}(x \mp d)^2$
  - This works but takes a long time
  - Competitive ratio $= O(d)$

V-Model

- Robots differ only in their time units, $v = 1$
- Reference frame trick does not work
  - Robots arrive to k^{th} turning points at different times
- Cannot use the algorithm $X_k = 2^k$
  - Cone of Alice: $\mathcal{T}(x) = 3|x| - 2$
  - Cone of Bob: $\mathcal{T}'(x) = 3|x \mp d| - 2\tau$
  - These may not contain each other
- Try the algorithm $\mathcal{T} = x^2$
  - $\mathcal{T}'(x) = \tau \left(\frac{x \mp d}{v\tau}\right)^2 = \frac{1}{\tau}(x \mp d)^2$
  - This works but takes a long time
  - Competitive ratio $= O(d)$
- We want a $\mathcal{T}(x)$ that is "in between" a linear and quadratic function...

V-Model

- Use the algorithm $X_k = (k+2)2^k$

[1] $W(x)$ is the W-Lambert function, i.e. inverse of $y = xe^x$

V-Model

- Use the algorithm $X_k = (k+2)2^k$
- Implies the cones[1]:
  - For Alice: $\mathcal{T}(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x|)}$
  - For Bob: $\mathcal{T}'(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x/\tau|)}$

[1] $W(x)$ is the W-Lambert function, i.e. inverse of $y = xe^x$

V-Model

- Use the algorithm $X_k = (k+2)2^k$
- Implies the cones[1]:
  - For Alice: $\mathcal{T}(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x|)}$
  - For Bob: $\mathcal{T}'(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x/\tau|)}$

Theorem

Rendezvous in V-Model can be solved with a competitie ratio $\frac{18\log^2(d)\tau}{|\log(\tau)|}$ + l.o.t..

[1] $W(x)$ is the W-Lambert function, i.e. inverse of $y = xe^x$

V-Model

- Use the algorithm $X_k = (k+2)2^k$
- Implies the cones[1]:
  - For Alice: $\mathcal{T}(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x|)}$
  - For Bob: $\mathcal{T}'(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x/\tau|)}$

Theorem

Rendezvous in V-Model can be solved with a competitie ratio $\frac{18\log^2(d)\tau}{|\log(\tau)|} +$ l.o.t..

- Competitive ratio still depends on d...

[1] $W(x)$ is the W-Lambert function, i.e. inverse of $y = xe^x$

V-Model

- Use the algorithm $X_k = (k+2)2^k$
- Implies the cones[1]:
  - For Alice: $\mathcal{T}(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x|)}$
  - For Bob: $\mathcal{T}'(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x/\tau|)}$

Theorem

Rendezvous in V-Model can be solved with a competitie ratio $\frac{18\log^2(d)\tau}{|\log(\tau)|} +$ l.o.t..

- Competitive ratio still depends on d...
- Is a constant competitive ratio possible?

[1] $W(x)$ is the W-Lambert function, i.e. inverse of $y = xe^x$

V-Model

- Use the algorithm $X_k = (k+2)2^k$
- Implies the cones[1]:
  - For Alice: $\mathcal{T}(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x|)}$
  - For Bob: $\mathcal{T}'(x) = 3|x| - \frac{4\ln(2)|x|}{W(4\ln(2)|x/\tau|)}$

Theorem

Rendezvous in V-Model can be solved with a competitie ratio $\frac{18\log^2(d)\tau}{|\log(\tau)|} +$ l.o.t..

- Competitive ratio still depends on d...
- Is a constant competitive ratio possible?

[1] $W(x)$ is the W-Lambert function, i.e. inverse of $y = xe^x$

V-Model

Is a constant competitive ratio possible?

- We don't think so...

V-Model

Is a constant competitive ratio possible?

- We don't think so...

Lemma

If $\mathcal{T}(x) = ax + O(1)$ then rendezvous may not occur.

V-Model

Is a constant competitive ratio possible?

- We don't think so...

Lemma

If $\mathcal{T}(x) = ax + O(1)$ then rendezvous may not occur.

Lemma

If $\mathcal{T}(x) = \omega(x)$ then the competitive ratio is $\omega(1)$.

V-Model

Is a constant competitive ratio possible?

- We don't think so...

Lemma

If $\mathcal{T}(x) = ax + O(1)$ then rendezvous may not occur.

Lemma

If $\mathcal{T}(x) = \omega(x)$ then the competitive ratio is $\omega(1)$.

General model

- No constraints on v and/or τ

General model

- No constraints on v and/or τ
- It is easy to show that the V-Model algorithm solves the problem
  - Use containment argument

General model

- No constraints on v and/or τ
- It is easy to show that the V-Model algorithm solves the problem
  - Use containment argument
- Worst case analysis $\rightarrow$ reduction to V-Model
  - If $v \neq 1$ rendezvous is “quick”

General model

- No constraints on v and/or τ
- It is easy to show that the V-Model algorithm solves the problem
  - Use containment argument
- Worst case analysis $\rightarrow$ reduction to V-Model
  - If $v \neq 1$ rendezvous is "quick"

Theorem

Rendezvous can be solved if at least one of v or τ is different than one.

Future directions

- Lower bounds and improvements on algorithms in V-Model
    - are improvements possible?

Future directions

- Lower bounds and improvements on algorithms in V-Model
    - are improvements possible?
- Extensions to the plane, 3D space, etc.

Future directions

- Lower bounds and improvements on algorithms in V-Model
    - are improvements possible?
- Extensions to the plane, 3D space, etc.
- Othe symmetric environments (e.g. cycle)

Future directions

- Lower bounds and improvements on algorithms in V-Model
    - are improvements possible?
- Extensions to the plane, 3D space, etc.
- Othe symmetric environments (e.g. cycle)

Thank you!