Large-Scale Distributed Algorithms for Facility Location with Outliers

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Joint work with Tanmay Inamdar and Sriram V. Pemmaraju

(Metric Uncapacitated) Facility Location

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Input:

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Objective:

$$\min_{S \subseteq F} \quad \sum_{i \in S} f_i + \sum_{j \in C} d(j, S)$$

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There are two sequential 3-approximation algorithms by –

- Jain and Vazirani (Primal-Dual Algorithm)
- Mettu and Plaxton (Greedy Algorithm)

Facility Location with Outliers

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How do we identify (and eliminate) outliers?

Facility Location with Outliers

Charikar et al. [1] proposed two variants of Facility Location with Outliers which we consider –

[1] Charikar, Khuller, Mount, Narasimhan: Algorithms for Facility Location Problems with Outliers, SODA 2001

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- Robust Facility Location
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Facility Location with Outliers

Charikar et al. [1] proposed two variants of Facility Location with Outliers which we consider –

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They also gave $O(1)$ approximation algorithms for both these problems.

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Robust Facility Location

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We are given a coverage requirement of $n - \ell$ clients.

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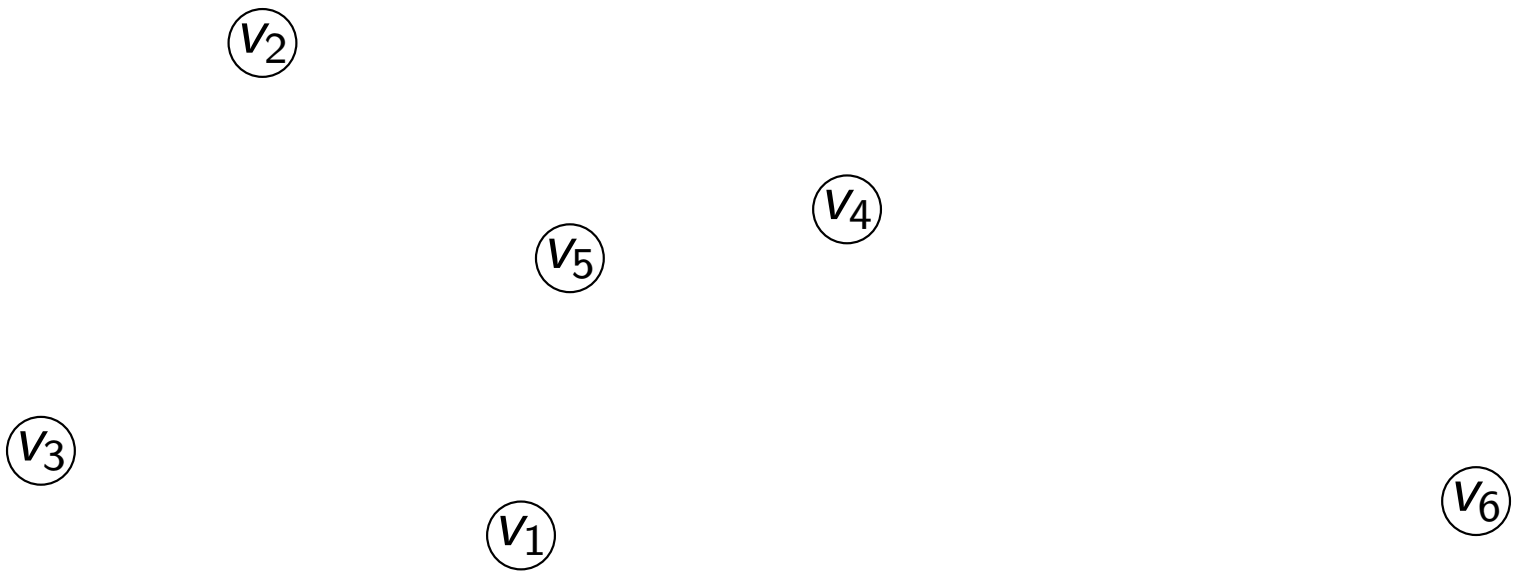
We want to minimize Facility Location objective while not serving at most ℓ clients (outliers).

An Example $\ell = 1$



Facilities and Clients are co-located

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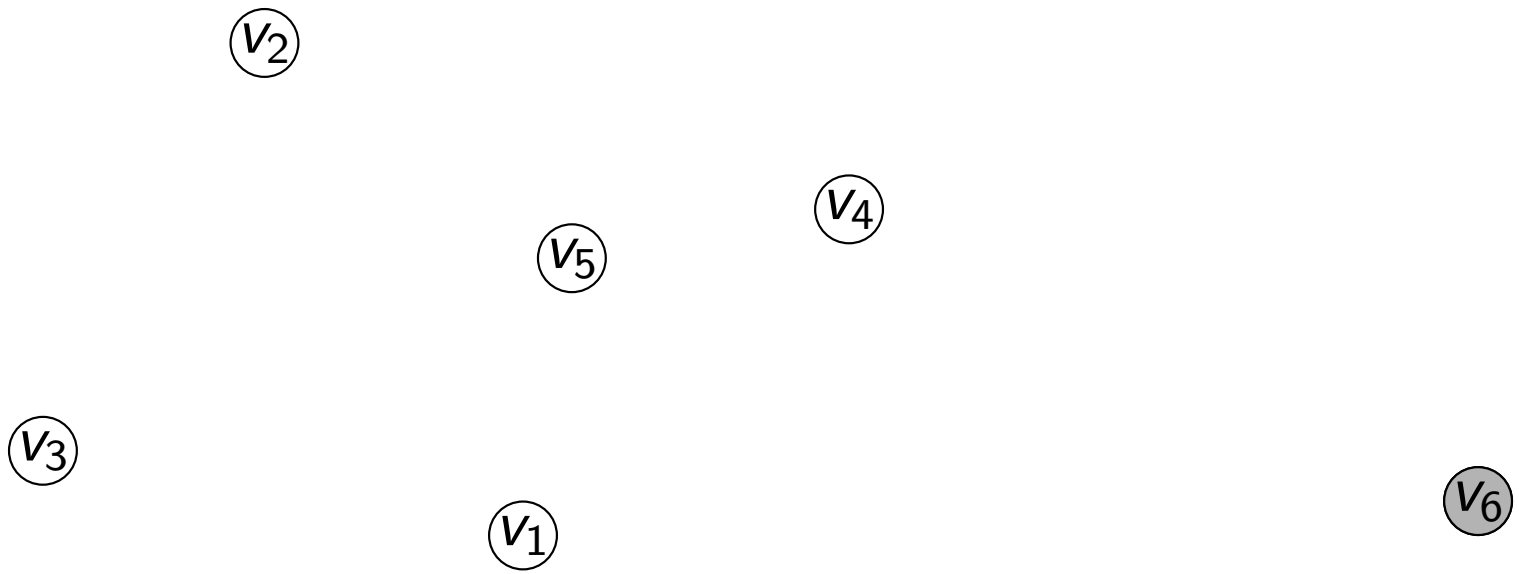

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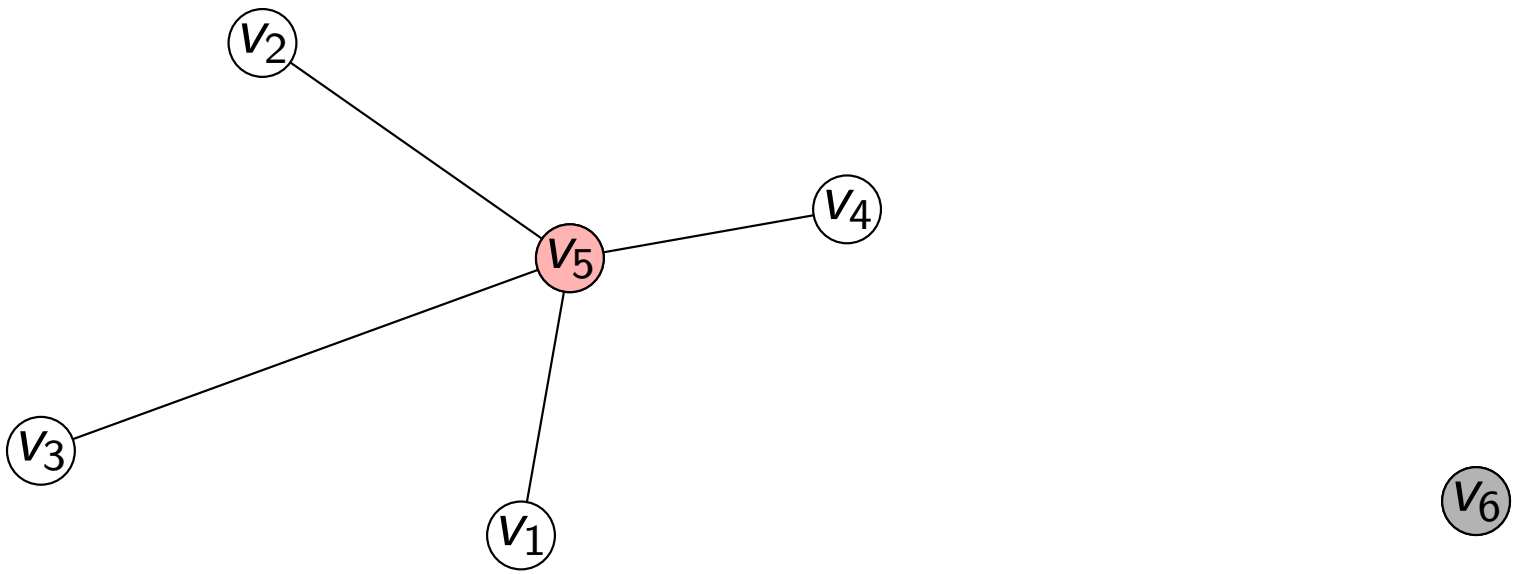

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In distributed settings, the complexity of the problem can be quite sensitive to the input representation.

We consider two alternate ways of specifying the input to the problem:

- **Explicit Metric:** The metric is specified as a $|F| \times |C|$ matrix distributed among the machines
- **Implicit Metric:** The metric is specified as the shortest path metric of a given edge-weighted graph whose vertex set is $F \cup C$

Why use Implicit Metric?

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Con: Need to perform shortest path computations to get distances between nodes.

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The Mettu-Plaxton algorithm allows much more efficient distributed implementations.

Main Technical Contribution

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We show that the Mettu-Plaxton algorithm can be modified in a similar manner to obtain $O(1)$ approximation algorithms for

- Robust Facility Location
- Facility Location with Penalties

Our Results

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We give constant approximation algorithms in three models of distributed computing:

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- Congested Clique Model
- Massively Parallel Computation (MPC) Model with $\tilde{O}(n)$ memory
- k-machine Model

Our Resuts

Round Complexity of our algorithms in different models:

	Implicit	Explicit
Congested Clique	$O(\text{poly} \log n)$	$O(\log \log \log n)$
MPC, $\tilde{O}(n)$ memory	$O(\text{poly} \log n)$	$O(\log \log \log n)$
k-machine	$\tilde{O}(n/k)$	$\tilde{O}(n/k)$

The Mettu-Plaxton Algorithm

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- **Radius Computation Phase.**
  - For each facility $v \in F$, compute radius r_v.
- **Greedy Phase.**
  - Consider facilities $v \in F$ in non-decreasing order of radii.
  - Starting with $S = \emptyset$, add v to S if $d(v, S) > 2r_v$.

Radius Definition

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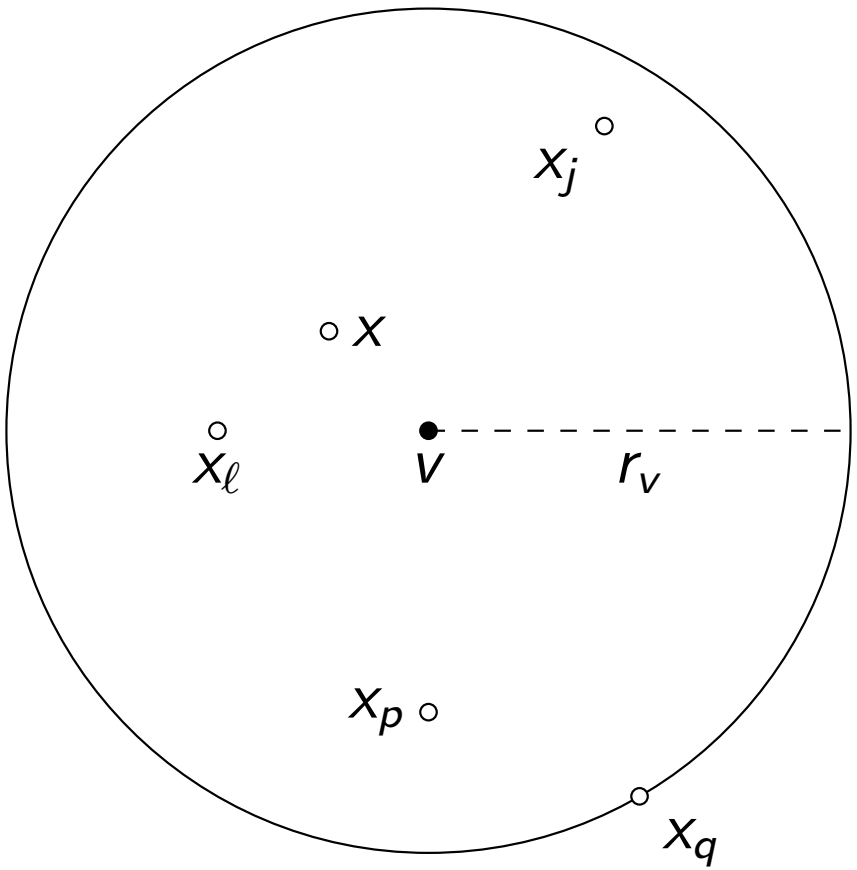

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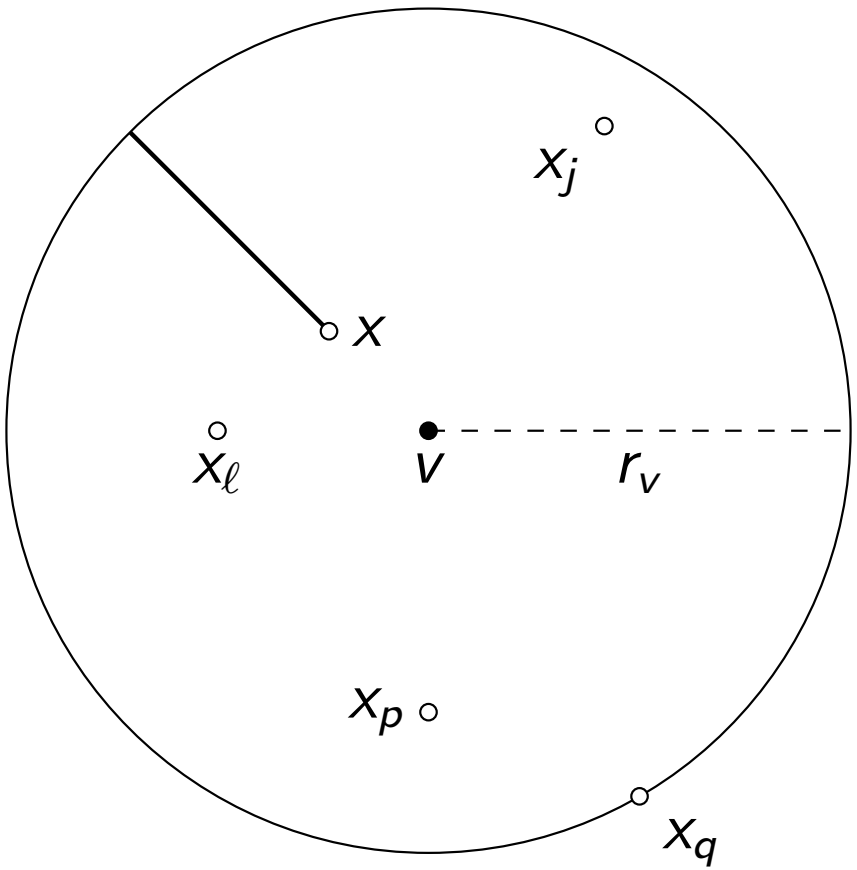

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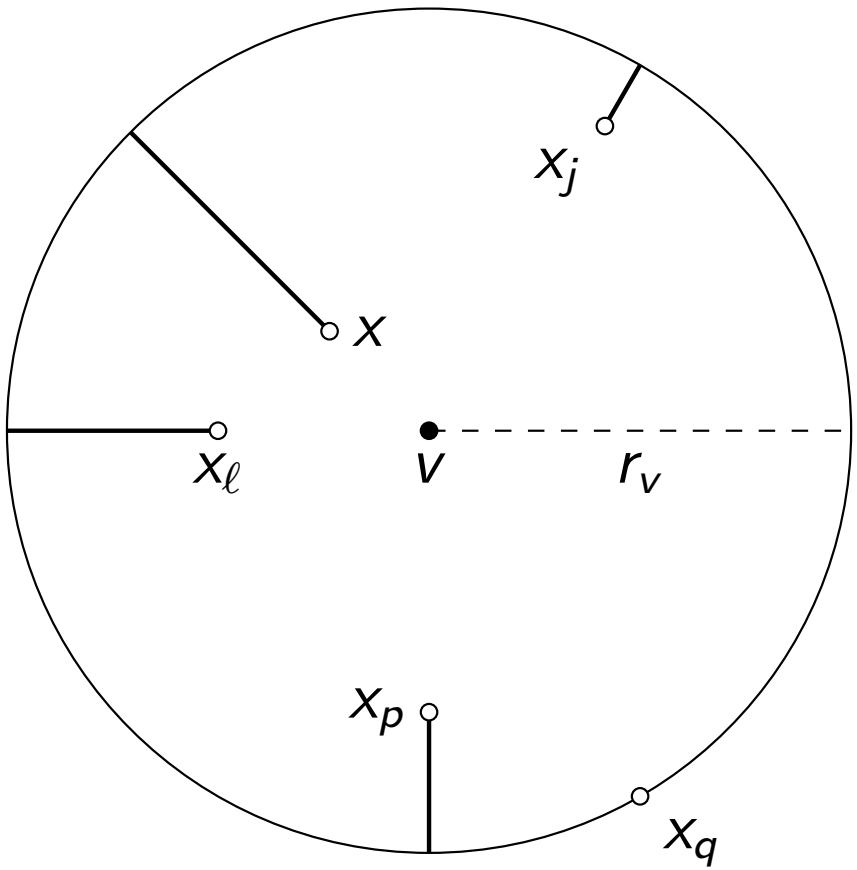

Radius Computation

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Implicit metric: radius computation can be done using polylog n calls to a $(1+\epsilon)$-approximate SSSP subroutine [2].

[2] Bandyapadhyay, Inamdar, Pai, Pemmaraju: Near-Optimal Clustering in the k-Machine Model, ICDCN 2018

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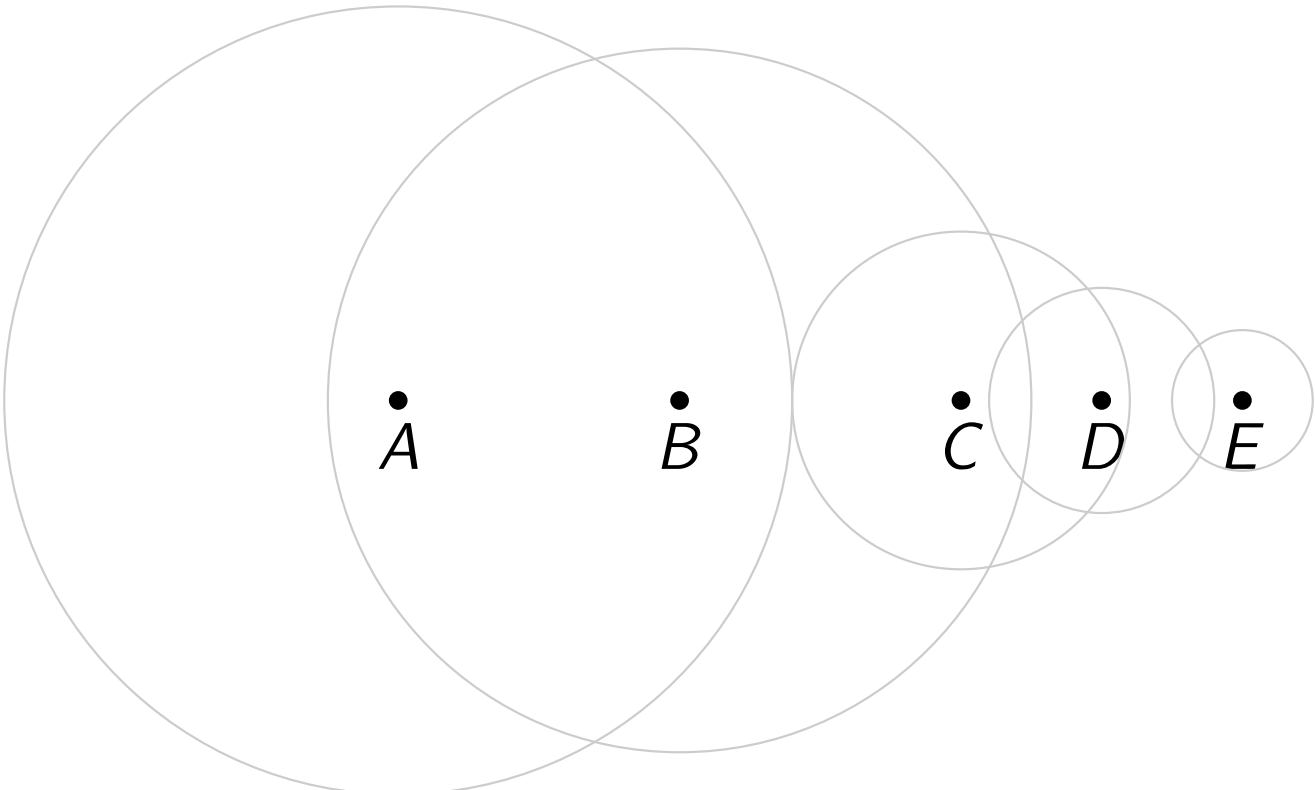

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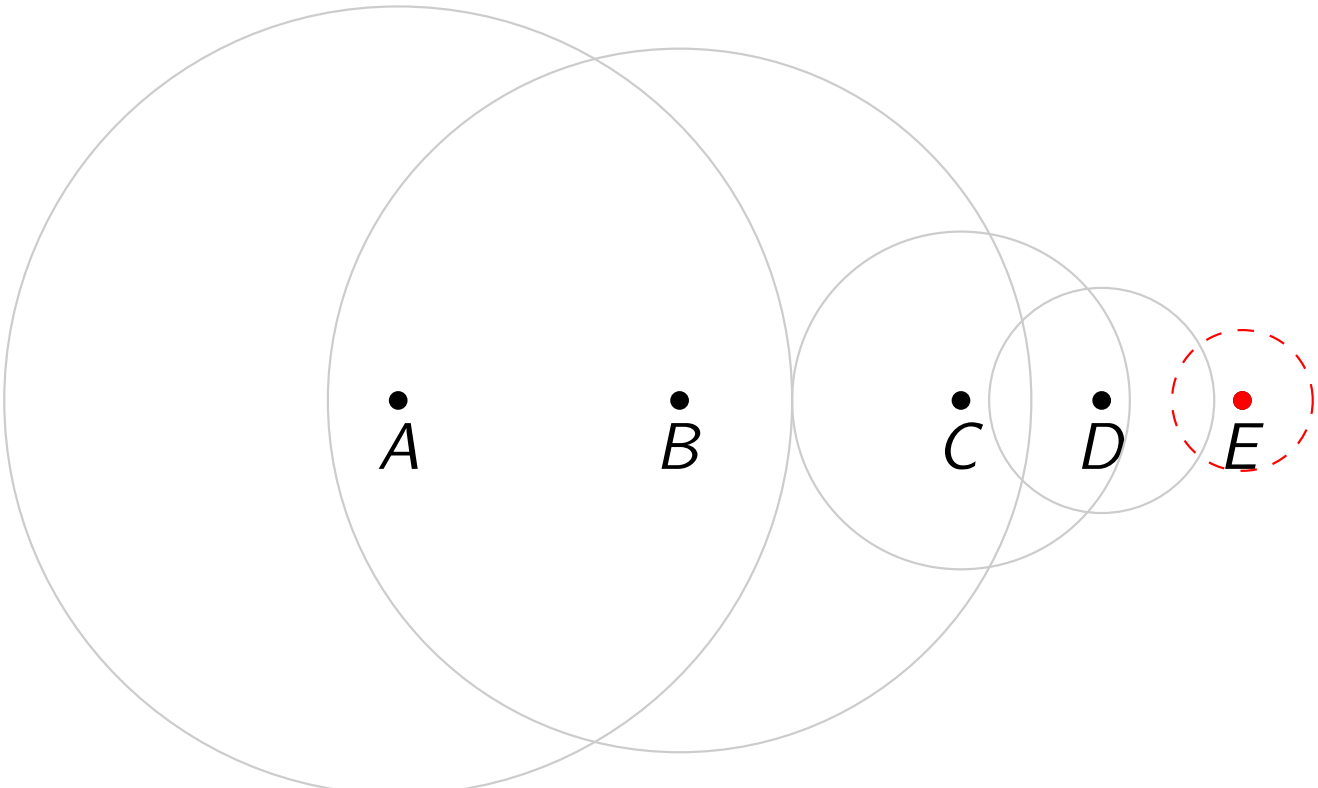

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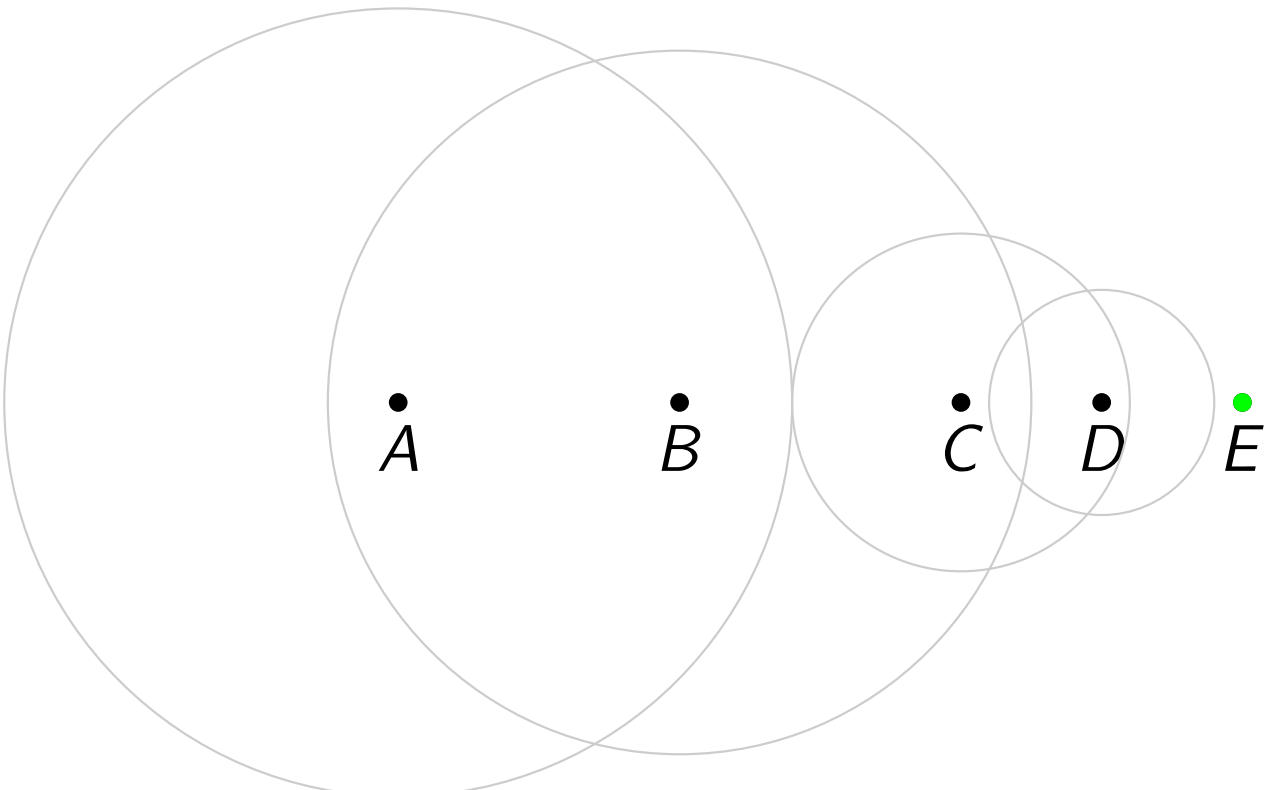

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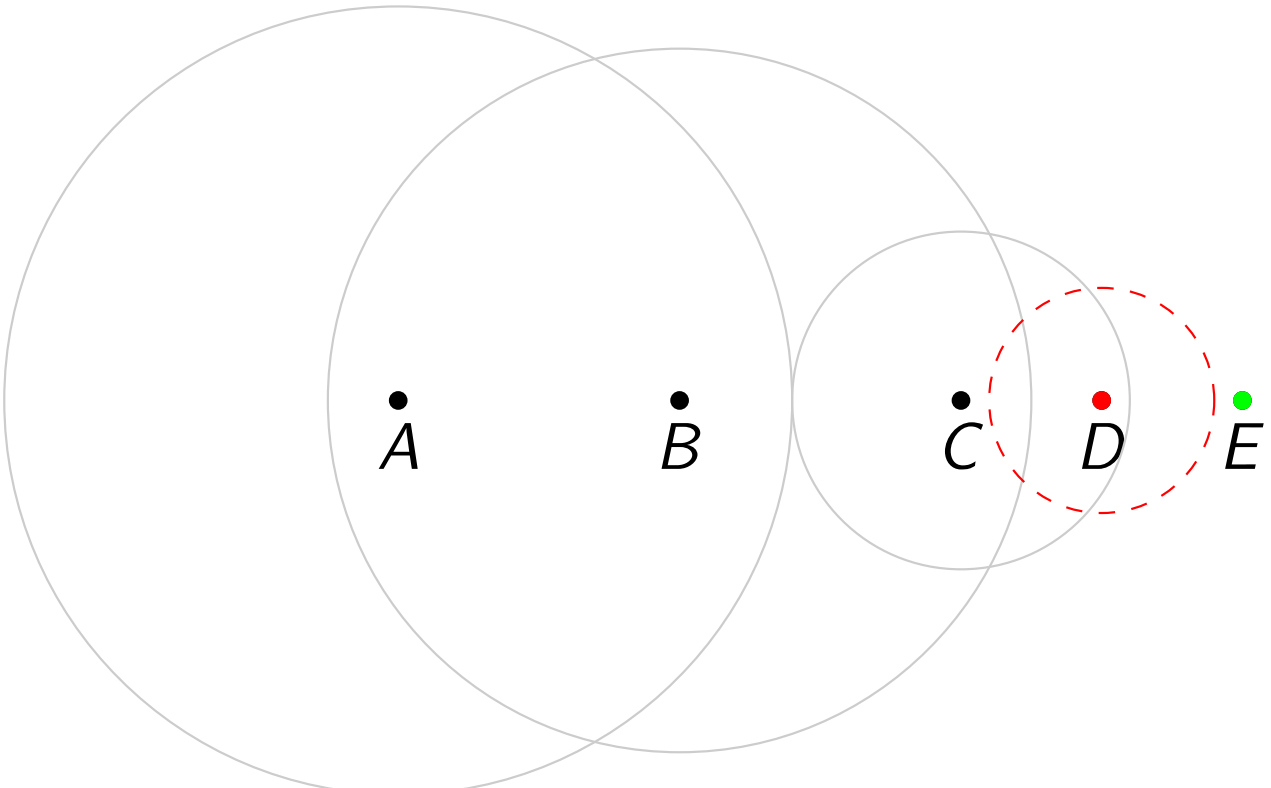

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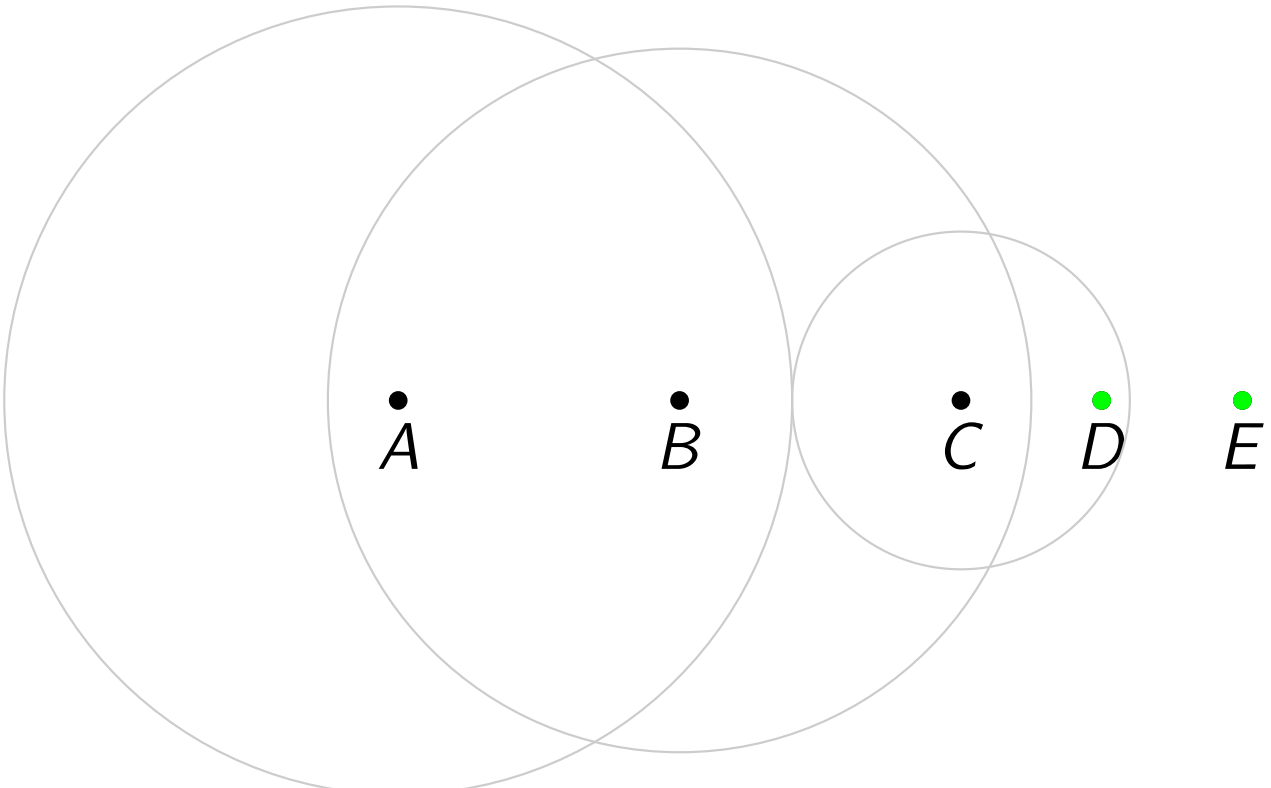

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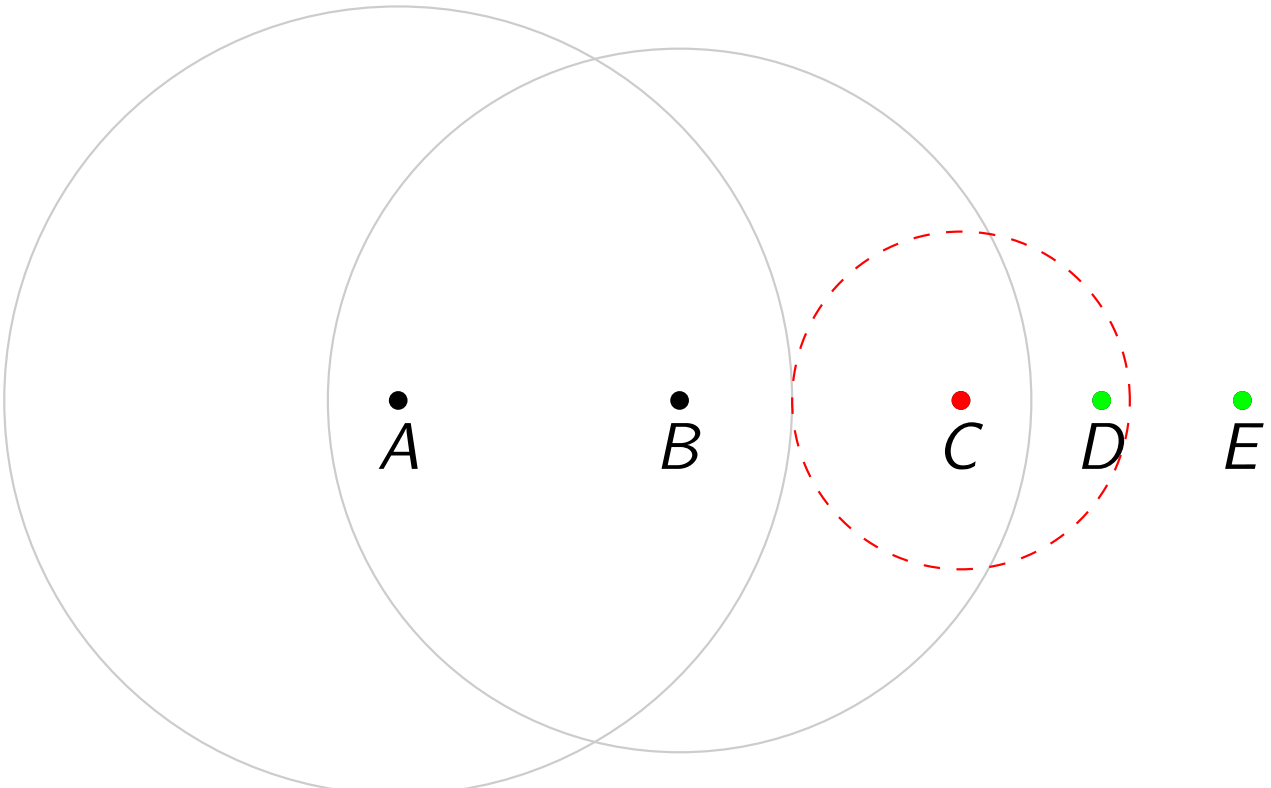

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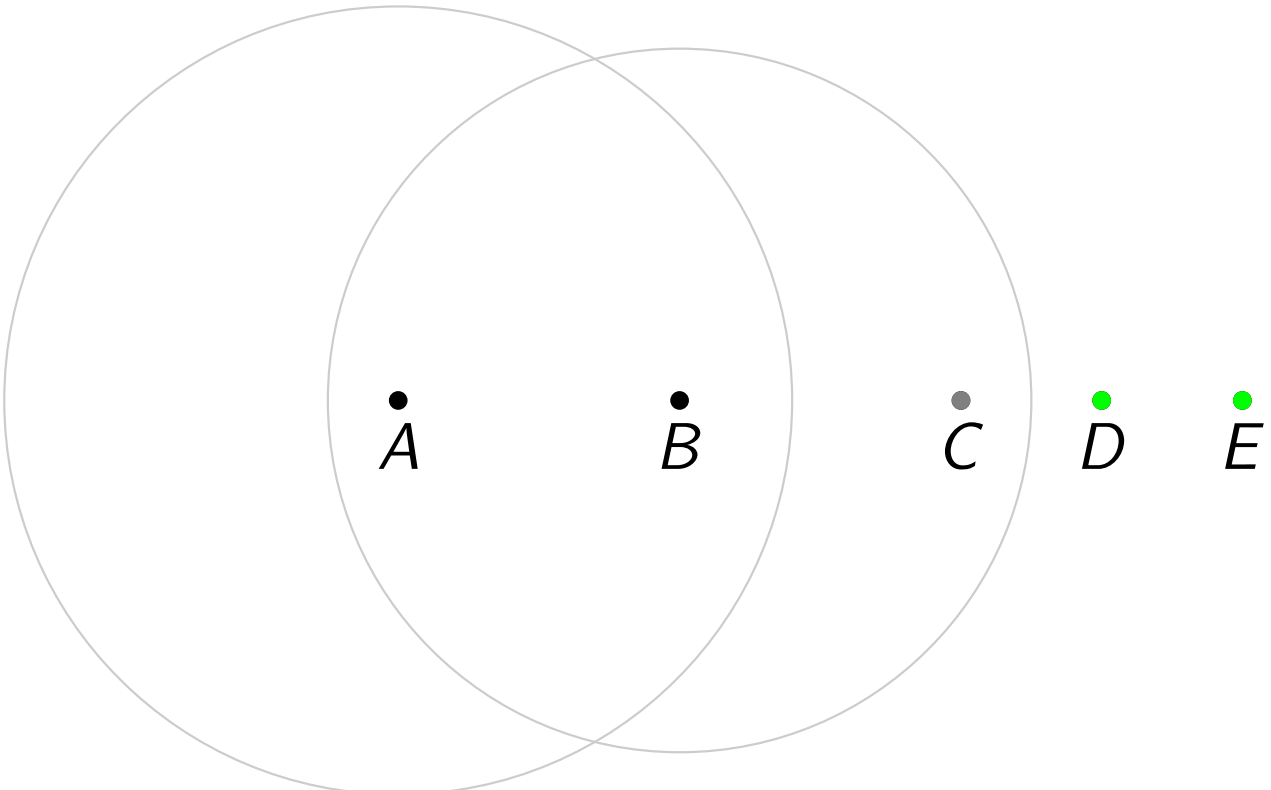

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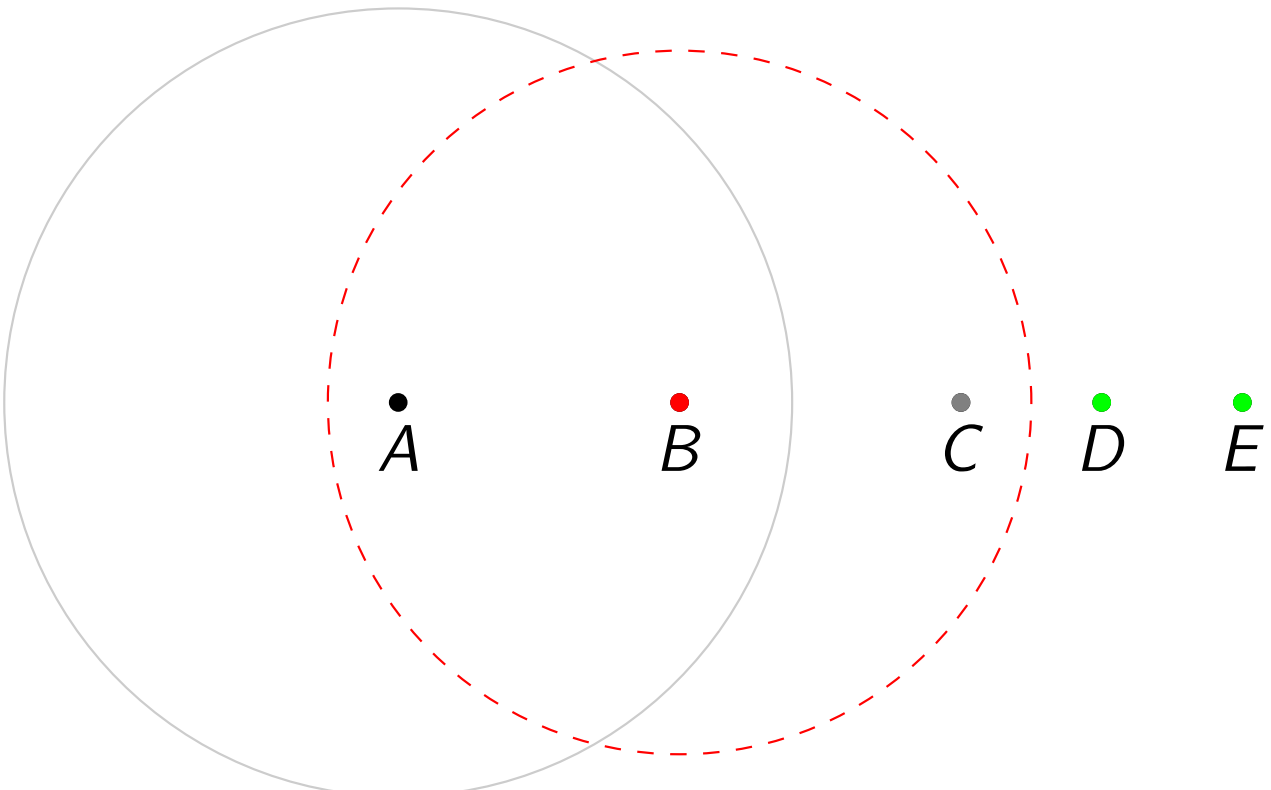

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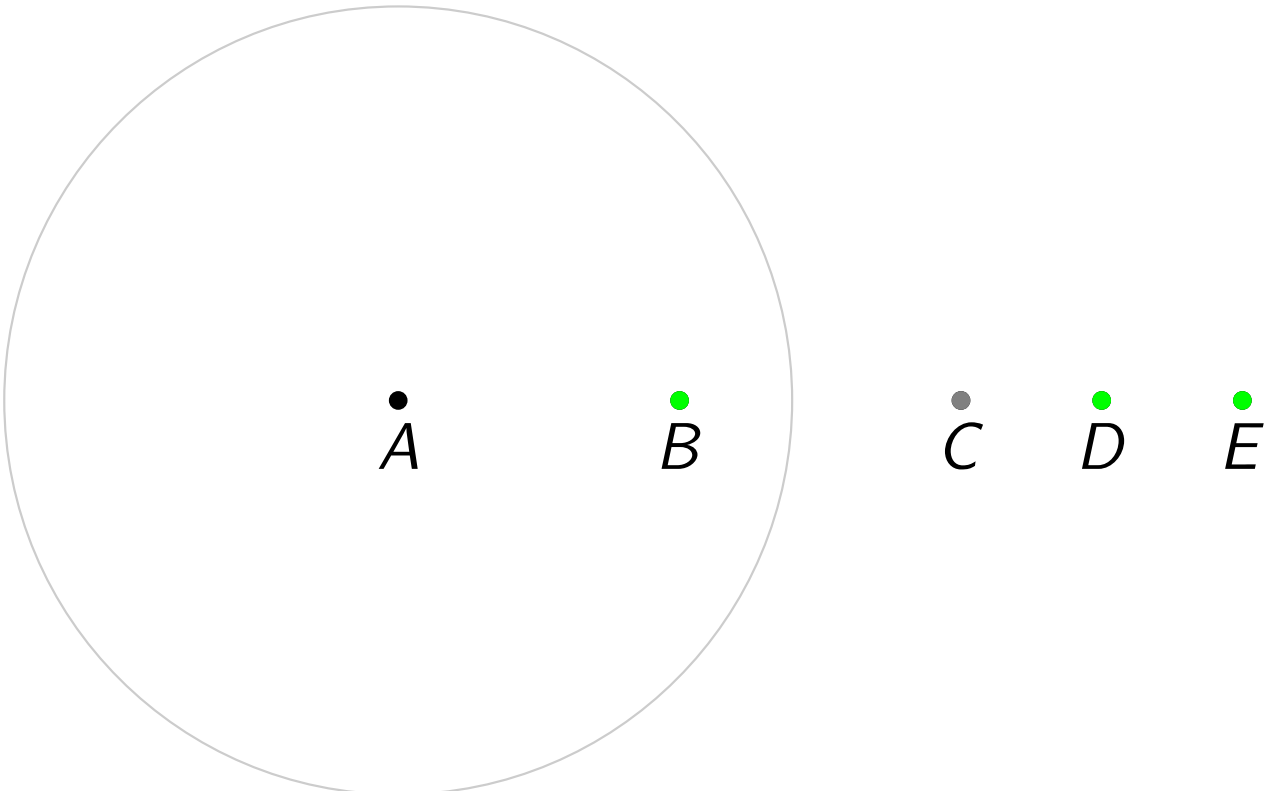

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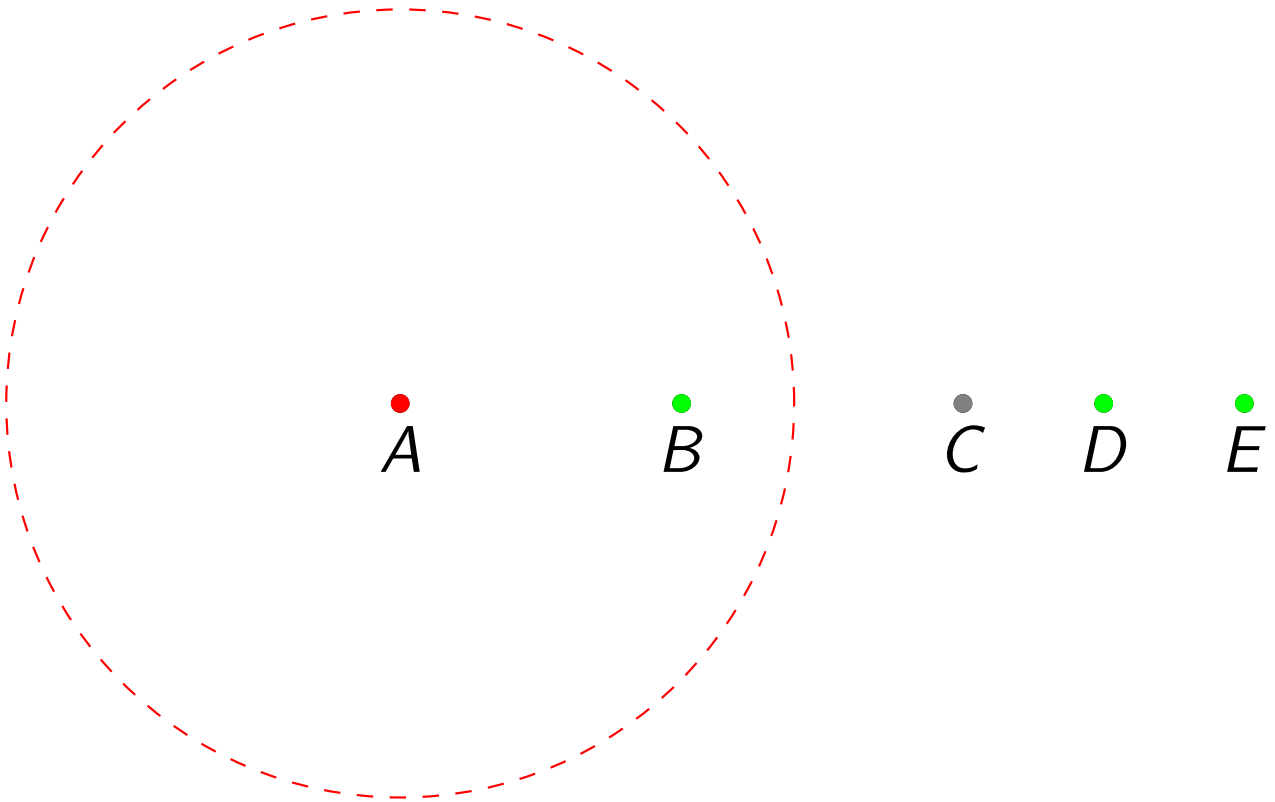

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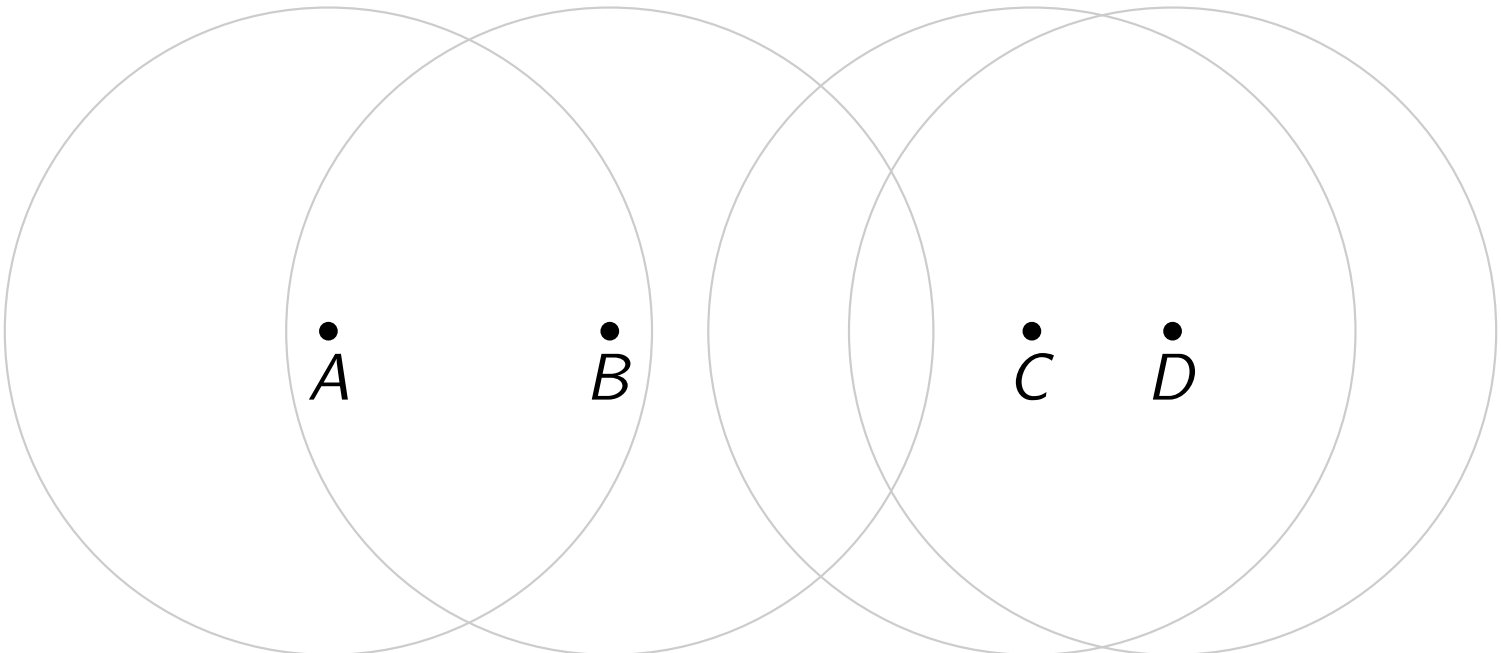

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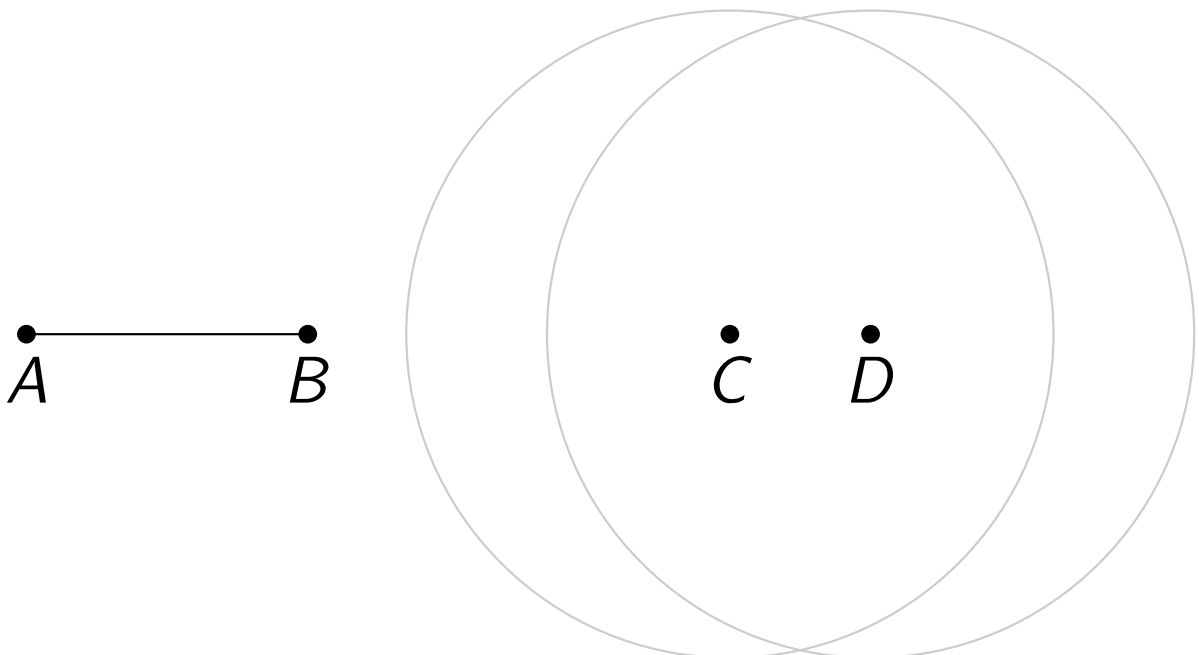

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We show how to compute a 2-ruling set in $O(\log \log \log n)$ rounds of Congested Clique with high probability.

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Running time is dominated by time required to learn relevant parts of the metric.

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SSSP can be solved in $O(\text{poly}(\epsilon^{-1})\log^3 n)$ rounds of Congested Clique [3].

[3] Becker, Karrenbauer, Krinninger, and Lenzen: Near-Optimal Approximate Shortest Paths and Transshipment in Distributed and Streaming Models, DISC 2017

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$$f_i' = \begin{cases} +\infty & \text{if } f_i > f_j \\ 0 & \text{if } i = j \\ f_i & \text{otherwise} \end{cases}$$

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Robust Facility Location —> $O(\log n)$ Facility Location computations

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- Works with the Mettu-Plaxton Algorithm.

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Robust Facility Location —> $O(\log n)$ Facility Location computations

- Works with the Mettu-Plaxton Algorithm.
- Can perform all $O(\log n)$ computations in parallel.

Future Directions

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- Can we get t-ruling set algorithms that run in $O(\log^* n)$ or $O(1)$ rounds of Congested Clique?

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- Can we get t-ruling set algorithms that run in $O(\log^* n)$ or $O(1)$ rounds of Congested Clique?
- Can we compute exact SSSP in $O(\operatorname{poly}\log n)$ rounds of Congested Clique?

Thank you

Questions?